

A HELPFUL MNEMONIC FOR TAKING DUALS: S.O.B.  
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In the table that follows, we provide a way to remember how to derive the dual from a linear program.

*The mnemonic is only that: a helpful way to remember the dual conversions. To understand WHY the dual conversions work the way they do, the theory of Lagrange can be used.*

We assume the linear program has been written so all the variables on the left-hand side of the constraints and the constants have been written on the right-hand side.

For example, the following LP is **NOT** in the correct form:

$$\begin{array}{ll} \min & 4x_1 + 3x_2 - 2x_3 \\ \text{s.t.} & x_1 + 2 = 7x_2 - 5x_3 \\ & 3 \geq x_2 \\ & x_1 \geq 8 + 7x_3 \\ & x_1 \geq 0, x_2 \leq 0 \end{array}$$

The form which makes the table work is as follows:

$$\begin{array}{ll} \min & 4x_1 + 3x_2 - 2x_3 \\ \text{s.t.} & 2x_1 - 7x_2 + 5x_3 = -2 \\ & x_2 \leq 3 \\ & x_1 - 7x_3 \geq 8 \\ & 5x_1 - x_2 \geq 0 \\ & x_1 \geq 0, x_2 \leq 0 \end{array}$$

The table works as follows. Recall that we think of each LP as having constraints and variable sign restrictions. Given the form that we write LPs in, we have the following constraint types:

1. **Less-then-or-equal-to:** Abbreviated with  $\leq$ , this refers to constraints where the variable sums are less than or equal to the constant, e.g.,

$$17x_1 - 5x_2 \leq 3.$$

2. **Greater-then-or-equal-to:** Abbreviated with  $\geq$ , this refers to constraints where the variable sums are greater than or equal to the constant, e.g.,

$$17x_1 - 5x_2 \geq 3.$$

3. **Equal-to:** Abbreviated with  $=$ , this refers to constraints where the variable sums are equal to the constant, e.g.,

$$17x_1 - 5x_2 = 3.$$

Variables are either nonnegative ( $\geq 0$ ), nonpositive ( $\leq 0$ ) or free ('free').

In a dual, each constraint in the primal is associated with a variable in the dual. Each variable in the primal is associated with a constraint in the dual. The idea behind Sensible, Odd, and Bizarre is as follows:

Sensible constraints go to sensible variables and vice versa, odd goes to odd, and bizarre goes to bizarre.

For the constraints, if we are maximizing (i.e., making the objective as much as possible), then a “sensible” constraint is if the amount the variable sum can be increased is restricted, i.e.,  $\leq$  is sensible. However, putting a limit on the amount we could reduce the sum is “bizarre”. If we were minimizing, then a sensible constraint would be to limit the amount we could reduce a sum, i.e.,  $\geq$  is sensible and limiting the amount we could increase would be “bizarre”. In both cases, an equality constraint is odd. For variables, nonnegative variables ( $\geq 0$ ) are sensible, nonpositive variables ( $\leq 0$ ) are bizarre, and free variables (‘free’) are *odd*.

The following table summarizes this memory device.

	Constraint types		Variable signs
	max	min	
<b>Sensible</b>	$\leq$	$\geq$	$\geq 0$
<b>Odd</b>	$=$	$=$	‘free’
<b>Bizarre</b>	$\geq$	$\leq$	$\leq 0$

Table 1: The **Sensible**, **Odd**, **Bizarre** mnemonic for duals

To illustrate this, here is the example again with its dual.

$$\begin{array}{ll}
 \min & 4x_1 + 3x_2 - 2x_3 \quad (a) \\
 \text{s.t.} & 2x_1 - 7x_2 + 5x_3 = -2 \quad (y_1) \\
 & x_2 \leq 3 \quad (y_2) \\
 & x_1 - 7x_3 \geq 8 \quad (y_3) \\
 & 5x_1 - x_2 \geq 0 \quad (y_4) \\
 & x_1 \geq 0, x_2 \leq 0 \quad (b)
 \end{array}
 \qquad
 \begin{array}{ll}
 \max & -2y_1 + 3y_2 + 8y_3 \quad (c) \\
 \text{s.t.} & 2y_1 + y_3 + 5y_4 \leq 4 \quad (x_1) \\
 & -7y_1 + y_2 - y_4 \geq 3 \quad (x_2) \\
 & 5y_1 - 7y_3 = -2 \quad (x_3) \\
 & y_2 \leq 0, y_3 \geq 0, y_4 \geq 0 \quad (d)
 \end{array}$$

The labels of the constraints of the primal and dual are the corresponding dual and primal variables. The objective in the dual, (c), has the right-hand sides of the constraints from the primal as coefficients. Also, as the primal is a minimization, the dual is a maximization. Note that  $y_4$  has a zero-coefficient, and so, does not appear in (c). The constraint in the dual associated with  $x_1$ , a nonnegative variable (sensible), must therefore be less-than-or-equal-to, as the objective is a maximization. The constraint associated with  $x_2$ , a nonpositive variable (bizarre), must be greater-than-or-equal-to, again, because the objective is a maximization problem. The last constraint, associated with  $x_3$ , a free variable (odd), must be an equality constraint. The primal constraint associated with the dual variable  $y_1$  is an equality (odd), therefore  $y_1$  is a free variable. The primal constraint associated with the dual variable  $y_2$  is less-than-or-equal-to in a minimization problem (bizarre), and therefore  $y_2$  is nonpositive. The primal constraint associated with  $y_3$  is greater-than-or-equal-to in a minimization problem (sensible), and therefore  $y_3$  is nonnegative. The primal constraint associated with  $y_4$  is greater-than-or-equal-to (sensible) so  $y_4$  is nonnegative.

*I learned this mneomic from Andrew M. Ross, currently a professor at Eastern Michigan State University in the Mathematics Department.*