Sunco formulation
SA305
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INDEX SETS

$j \in \mathcal{G}$ = set of gasoline types;
$i \in \mathcal{C}$ = set of crude oil types.

PARAMETERS

p_i = price of crude oil i per barrel in dollars;
r_j = revenue in dollars from one barrel of gasoline j;
s_i = percentage sulphur content of crude i;
σ_j = upper bound on the percentage of sulphur in gasoline j;
t_i = octane rating of crude i;
τ_j = lower bound on the octane rating of gasoline j;
d_j = obligatory demand for gasoline j.

DECISION VARIABLES

g_j = barrels of gasoline j to produce;
c_i = barrels of crude i to buy;
x_{ij} = barrels of crude i blended to produce gasoline j;
a_j = \$ spent on advertising gasoline j.

FORMULATION

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\begin{array}{lll}
\max & \sum_{j\in\mathcal{G}}(r_j-4)g_j - \sum_{i\in\mathcal{C}} p_i c_i - \sum_{j\in\mathcal{G}} a_j & \text{(a)} \\
\text{s.t.} & g_j = \sum_{i\in\mathcal{C}} x_{ij} \qquad \forall j \in \mathcal{G} & \text{(b)} \\
& c_i = \sum_{j\in\mathcal{G}} x_{ij} \qquad \forall i \in \mathcal{C} & \text{(c)} \\
& \sum_{j\in\mathcal{G}} g_j \le 14000 & \text{(d)} \\
& g_j \ge d_j \qquad \forall j \in \mathcal{G} & \text{(e)} \\
& g_j \le d_j + a_j \qquad \forall j \in \mathcal{G} & \text{(f)} \\
& \sum_{i\in\mathcal{C}} s_i x_{ij} \le \sigma_j g_j \qquad \forall j \in \mathcal{G} & \text{(g)} \\
& \sum_{i\in\mathcal{C}} t_i x_i \ge \tau_j g_j \qquad \forall j \in \mathcal{G} & \text{(h)} \\
& g_j, c_i, x_{ij}, a_j \ge 0 \qquad \forall i \in \mathcal{C}, \forall j \in \mathcal{G} & (i)
\end{array}
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DISCUSSION The objective, (a), represents the profit from a given setting of gas production, blend composition, and advertising taking into account advertising costs, crude oil purchases, gas sale revenues, and crude oil processing costs. The calculation of each gas type produced and each crude oil type purchased are represented by (b) and (c). The constraint (d) enforces the limit on the amount of gas the refinery can process. Constraint (e) ensures enough gas of each type is produced and constraint (f) ensures that gas production does not exceed demand for each type. Note that constraint (e) and (f) assume advertising demand is *optional*, i.e., does not need to be satisfied. If the advertising demand created were obligatory rather than optional then a_j should be added to d_j for each gas type $j \in \mathcal{G}$ in constraint (e) and constraint (f) would be omitted. Constraints (f) and (g) are the blending constraints on sulpher and octane for each gas type, respectively. Constraint (i) ensures nonnegativity for all decision variables.