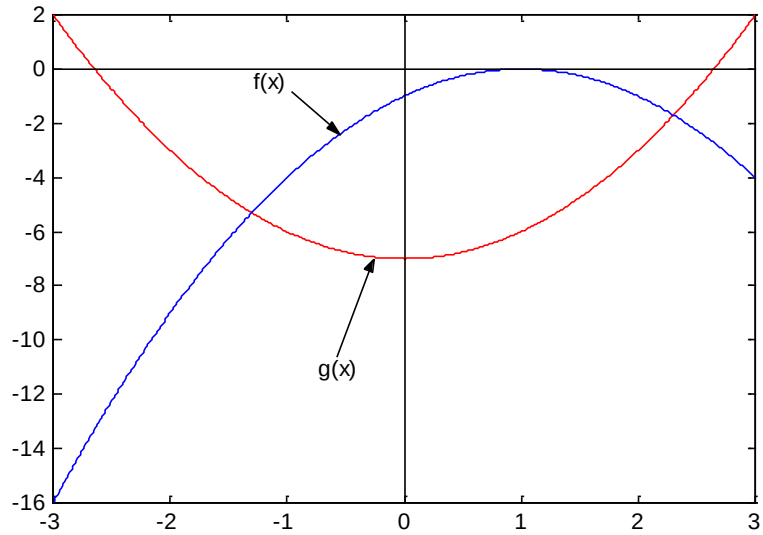
Calculus II – Exam 2 – Review Sheet

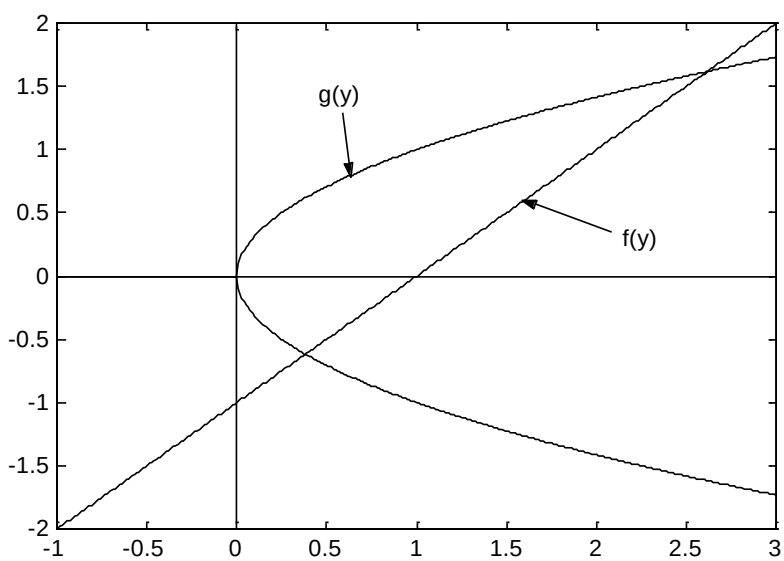
LCDR Zielinsky (Ret. U.S.N.) – Assoc. Prof. Liakos

Areas between 2 Curves:

$$\int_a^b (Top\ Curve - Bottom\ Curve)dx = \int_a^b [f(x) - g(x)]dx$$



$$\int_c^d (Right\ Curve - Left\ Curve)dy = \int_c^d [f(y) - g(y)]dy$$



Volumes of Revolution

Disks:

$\int_a^b \pi r^2 dx$ → for curves rotated around a horizontal line

$\int_c^d \pi r^2 dy$ → for curves rotated around a vertical line

Washers:

$\int_a^b \pi(r_{outer}^2 - r_{inner}^2)dx$ → for curves rotated around a horizontal line

$\int_c^d \pi(r_{outer}^2 - r_{inner}^2)dy$ → for curves rotated around a vertical line

Shells:

$$\int_a^b 2\pi r_{avg} h(thickness)$$

Thickness = dx for curves rotated around a vertical line
Thickness = dy for curves rotated around a horizontal line

Applications

Work

$$W = \int_a^b F(x)dx$$

- For springs, $F = kx$ (Hooke's Law).

- For lifting cables/ropes, $F =$(weight/length of rope or chain) x , where x is units from the top. If there is a weight attached to the rope or chain, add it to F.

- For draining pools, draw a "slice" of water x units from the top of the pool; then $F = \rho g x$ (length)(width),where ρg is 62.5 (for feet, pounds) or 9800 (in SI units).

  If the water needs to be drained from a spout s units above the top of the pool, replace x by $x + s$ above.

  Typically, the (length) is the challenging quantity to compute. For pools with straight edges, use similar triangles. For pools that are round use the Pythagorean Theorem on the right triangle formed by drawing the radius that connects the center to an endpoint of the slice.

Fluid Pressure/Hydrostatic Force

$$F = \int_a^b \rho g(depth)(length)dx$$

Draw a strip on the submerged plate x units from the top of the plate; ρg is 62.5 (for feet, pounds) or 9800 (in SI units).

Typically, the (length) is the challenging quantity to compute. For plates with straight edges, use similar triangles. For plates that are round use the Pythagorean Theorem on the right triangle formed by drawing the radius that connects the center to an endpoint of the strip.

Center of Mass

Point Mass

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n}$$

$$\bar{y} = \frac{m_1y_1 + m_2y_2 + \cdots + m_ny_n}{m_1 + m_2 + \cdots + m_n}$$

The center of mass of a plate of uniform density (with three straight edges) is located at the geometric center:

$$\bar{x} = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}[f(x)]^2dx}{\int_a^b f(x)dx}$$

If the plate is prescribed by two curves (i.e. it does not have three straight edges) then

$$\bar{x} = \frac{\int_a^b x[f(x) - g(x)]dx}{\int_a^b f(x)dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}\{[f(x)]^2 - [g(x)]^2\}dx}{\int_a^b f(x)dx}$$