Solutions to Homework #4

1

(c) To ten digits $\pi \ln 2 + \sqrt{10}\cos 22^\circ = 5.109598880$. In chopping arithmetic, we first calculate

$$\begin{aligned} \pi \times_{fl} \ln 2 &= fl_{\text{chop}}(fl_{\text{chop}}(\pi) \times fl_{\text{chop}}(\ln 2)) = fl_{\text{chop}}(3.141 \times 0.6931) \\ &= fl_{\text{chop}}(2.1770271) = 2.177 \end{aligned}$$

and

$$\begin{aligned} \sqrt{10} \times_{fl} \cos 22^\circ &= fl_{\text{chop}}(fl_{\text{chop}}(\sqrt{10}) \times fl_{\text{chop}}(\cos 22^\circ)) \\ &= fl_{\text{chop}}(3.162 \times 0.9271) \\ &= fl_{\text{chop}}(2.9314902) = 2.931 \end{aligned}$$

Finally,

$$\begin{aligned} (\pi \times_{fl} \ln 2) +_{fl} (\sqrt{10} \times_{fl} \cos 22^\circ) &= fl_{\text{chop}}(fl_{\text{chop}}(\pi \ln 2) + fl_{\text{chop}}(\sqrt{10}\cos 22^\circ)) \\ &= fl_{\text{chop}}(2.177 + 2.931) \\ &= fl_{\text{chop}}(5.108) = 5.108. \end{aligned}$$

In rounding arithmetic, we calculate

$$\begin{aligned} \pi \times_{fl} \ln 2 &= fl_{\text{round}}(fl_{\text{round}}(\pi) \times fl_{\text{round}}(\ln 2)) \\ &= fl_{\text{round}}(3.142 \times 0.6931) \\ &= fl_{\text{round}}(2.1777202) = 2.178 \end{aligned}$$

and

$$\begin{aligned} \sqrt{10} \times_{fl} \cos 22^\circ &= fl_{\text{round}}(fl_{\text{round}}(\sqrt{10}) \times fl_{\text{round}}(\cos 22^\circ)) \\ &= fl_{\text{round}}(3.162 \times 0.9272) \\ &= fl_{\text{round}}(2.9318064) = 2.932 \end{aligned}$$

Finally,

$$\begin{aligned} (\pi \times_{fl} \ln 2) +_{fl} (\sqrt{10} \times_{fl} \cos 22^\circ) &= fl_{\text{round}}(fl_{\text{round}}(\pi \ln 2) + fl_{\text{round}}(\sqrt{10}\cos 22^\circ)) \\ &= fl_{\text{round}}(2.178 + 2.932) \\ &= fl_{\text{round}}(5.110) = 5.110. \end{aligned}$$

Absolute and relative errors in both the chopped and the rounded values are given in the table below.

In the following table, δ denotes the absolute error and ϵ the relative error.

	Chopping		Rounding	
	value	error	value	error
(c)	5.108	$\delta = 1.599 \times 10^{-3}$ $\epsilon = 3.129 \times 10^{-4}$	5.110	$\delta = 4.011 \times 10^{-4}$ $\epsilon = 7.850 \times 10^{-5}$

2

There is the possibility for cancellation error in STEP 1 (if $b^2 \approx 4ac$) and in STEP 2 (if $b \approx disc$). There is the possibility for amplification of roundoff error in STEP 3 (if $root1 \approx 0$).

7

(a) Because

$$\left| \frac{\sqrt{10002} - \sqrt{10001}}{\sqrt{10001}} \right| = 4.999 \times 10^{-5}$$

and

$$10^{-5} < 4.999 \times 10^{-5} \leq 10^{-4},$$

it follows that $\sqrt{10002}$ and $\sqrt{10001}$ agree to at least 4 and at most 5 decimal digits.

(b) In $\mathbf{F}(10, 10, -98, 100)$,

$$\sqrt{10002} = 100.0099995 \qquad \text{and} \qquad \sqrt{10001} = 100.0049999.$$

Therefore,

$$\sqrt{10002} - \sqrt{10001} = 100.0099995 - 100.0049999 = 4.9996 \times 10^{-3}.$$

Because each operand has ten significant decimal digits while the result has only five, five significant decimal digits have been lost in performing the subtraction.

(c) To avoid subtracting two nearly equal numbers, we rewrite the expression as

$$\sqrt{10002} - \sqrt{10001} = \frac{1}{\sqrt{10002} + \sqrt{10001}}.$$

Now, working in $\mathbf{F}(10, 10, -98, 100)$, we find

$$\sqrt{10002} - \sqrt{10001} = \frac{1}{\sqrt{10002} + \sqrt{10001}} = 4.999625043 \times 10^{-3}.$$

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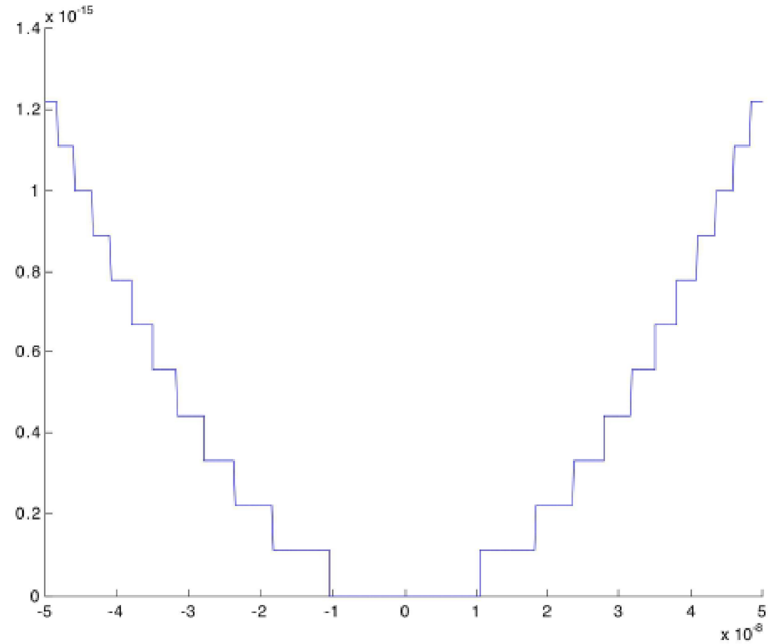
(a) Here is a graph of $f(x) = 1 - \cos x$ over the interval $-5\times 10^{-8} \le x \le 5\times 10^{-8}$. All calculations were performed in IEEE standard double precision.



(b) Starting from the identity $\cos 2x = 1 - 2\sin^2 x$, we find $1 - \cos 2x = 2\sin^2 x$. Therefore, $f(x) = 1 - \cos x = 2\sin^2(x/2)$. The following graph displays $2\sin^2(x/2)$ over the interval $-5\times 10^{-8} \le x \le 5\times 10^{-8}$. All calculations were performed in IEEE standard double precision.