Three Versions of the Fundamental Theorem of Calculus

1-d. **Fundamental Theorem of Calculus for Line Integrals:** for an oriented curve C having starting point p and end point q,

$$\int_C \nabla f \cdot d\vec{r} = f(q) - f(p).$$

Consequence: if $\vec{F}$ is conservative, then $\int_C \vec{F} \cdot d\vec{r} = 0$ for any closed curve C.

2-d. **Stokes's Theorem:** For a surface Σ having boundary curve C,

$$\iint_\Sigma \nabla \times \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r},$$

provided the normal vector to Σ and the orientation of C are consistent via the right-hand rule.
Consequences: a. Green's Theorem, that is, if the surface Σ is in the xy-plane, then

$$\iint_\Sigma \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA = \int_C \vec{F} \cdot d\vec{r}.$$

b. If $\vec{G} = \nabla \times \vec{F}$ for some vector field $\vec{F}$, then $\iint_S \vec{G} \cdot d\vec{S} = \iint_\Sigma \vec{G} \cdot d\vec{S}$ for any two surfaces S, Σ with the same boundary curve.

3-d. **Divergence Theorem:** For a solid E with boundary surface Σ,

$$\iiint_E \nabla \cdot \vec{F} \, dV = \iint_\Sigma \vec{F} \cdot d\vec{S},$$

provided an outward normal is used on the right-hand side.
Consequence: if $\vec{F}$ is incompressible (that is, if $\nabla \cdot \vec{F} = 0$) then $\iint_S \vec{F} \cdot d\vec{S} = 0$ for any closed surface S.