Fall 2011. SM 223. Practice Test 3

This is a practice test. No claim is made that the real test will have the same or similar problems on it. The only claim is that the practice test covers the same concepts and topics as the real test. **Have you done and reviewed the homework? You probably should do that first.**

1. Let $f(x, y, z) = e^{x^2-y^2} - z^2$.

   (a) Find the tangent plane to the level surface $f(x, y, z) = 0$ at the point $(2, -2, 1)$. Write your answer in the form $z = ax + by + c$ for some constants a, b, c.

   (b) Use this or any other method to find the linear approximation of the function $z = g(x, y) = e^{(x^2-y^2)/2}$ at the point $(2, -2)$.

   (c) Use this linear approximation to estimate the value of $g(2.01, -1.98)$.

2. Suppose that $z = f(x, y)$ and that $x = s^2 - t^2$, $y = s^2 + t^2$ for some function f. Use the table below to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $s = 1$, $t = -2$.

(x, y)	$f(x, y)$	$f_x(x, y)$	$f_y(x, y)$	$f_{xx}(x, y)$	$f_{yy}(x, y)$	$f_{xy}(x, y)$
$(1, -2)$	1	2	3	4	5	6
$(-3, 5)$	7	8	9	10	11	12
$(2, -4)$	13	14	15	16	17	18

3. Find the local maxima and minima for the function $f(x, y) = xy + \dfrac{1}{x} + \dfrac{1}{y}$. For each critical point determine whether it is a local max, local min or saddle point and give the value of the function at that point. Present your answer clearly.

4. Find the maximum and minimum values of the functions $f(x, y, z) = x^3y^3z^3$ subject to the constraint $x^2 + y^2 + z^2 = 3$. Give all points at which the maximum and minimum value is attained and compute the maximum and minimum values.

5. Find the maximum volume of an open-top box whose surface area is 1 m^2.