Fall 2011. SM 223. Practice Test 4

This is a practice test. No claim is made that the real test will have the same or similar problems on it. The only claim is that the practice test covers the same concepts and topics as the real test. **Have you done and reviewed the homework? You probably should do that first.**

1. Use a double integral to compute the volume of the region bounded by the surface $z = y^2$, $y = 0$, $x = 0$ and $x + y = 1$.

   ———

   The region can be described by $0 \le x \le 1$, $0 \le y \le 1 - x$ and $0 \le z \le y^2$. The double integral is given by
   $$\int_0^1 \int_0^{1-x} y^2 \, dy \, dx.$$
   The inner integral is
   $$\int_0^{1-x} y^2 \, dy = \frac{(1-x)^3}{3}.$$
   The outer integral is
   $$\frac{1}{3} \int_0^1 (1-x)^3 \, dx = \frac{1}{12}.$$

2. Compute the double integral of the function $f(x, y) = xy$ over the quadrilateral with vertices $(0, 0)$, $(2, 1)$, $(2, 2)$ and $(0, 1)$.

   ———

   The quadrilateral can be described by the bounds $0 \le x \le 2$, $\frac{x}{2} \le y \le \frac{x}{2} + 1$.

   The inner integral is
   $$\int_{x/2}^{x/2+1} xy \, dy = \frac{xy^2}{2}\bigg|_{x/2}^{x/2+1} = \frac{x}{2}((x/2+1)^2 - (x/2)^2) = \frac{x}{8}(4x+4) = \frac{x}{2}(x+1)$$
   The outer integral is
   $$\int_0^2 \frac{x}{2}(x+1) \, dx = \frac{1}{2}\int_0^2 x^2 + x \, dx \; \frac{1}{2}(\frac{x^3}{3} + \frac{x^2}{2})\bigg|_0^2 = \frac{1}{2}(8/3 + 2) = \frac{7}{3}.$$

3. By interchanging the order of integration compute $\int_0^1 \int_x^1 e^{x/y} \, dy \, dx$

   ———

   The region is the triangle bounded by $0 \le x \le 1$ and $x \le y \le 1$. This can be described by $0 \le y \le 1$ and $0 \le x \le y$.

   We have
   $$\int_0^y e^{x/y} \, dx = ye^{x/y}\bigg|_0^y = ey - y = (e-1)y.$$
   The outer integral
   $$\int_0^1 (e-1)y \, dy = \frac{(e-1)}{2}.$$

4. Evaluate $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sin(x^2+y^2)\,dy\,dx$ by switching to polar coordinates.

The region is the half-disk of radius 3 that lies above the x-axis. In polar coordinates this disk is given by $0 \le \theta \le \pi$ and $0 \le r \le 3$.

The integral becomes

$$\int_0^3 \int_0^\pi \sin(r^2)\, r\, dr\, d\theta = \int_0^\pi \int_0^3 r\sin(r^2)\, dr\, d\theta.$$

The inner integral is given by

$$\int_0^3 r\sin(r^2)\, dr = -\frac{1}{2}\cos(r^2)|_0^3 = \frac{1}{2}(1-\cos(9))$$

The outer integral is

$$\int_0^\pi \frac{1}{2}(1-\cos(9))\, d\theta = \frac{\pi}{2}(1-\cos(9))$$

5. Compute the triple integral $\iiint_E xyz\, dV$, where E is the region bounded by $y=0$, $y=x^2$, $x=1$, and the planes $z=0$ and $z=x+y$.

The region is given by $0 \le x \le 1$, $0 \le y \le x^2$, $0 \le z \le x+y$.

Inner integral

$$\int_0^{x+y} xyz\, dz = \frac{xy}{2}(x+y)^2$$

The middle integral

$$\int_0^{x^2} \frac{xy}{2}(x+y)^2\, dy = \int_0^{x^2} \frac{1}{2}(x^3y+2x^2y^2+y^3x)\, dy = \left.\frac{6x^3y^2+8x^2y^3+y^4x}{24}\right|_0^{x^2} = \frac{6x^7+8x^8+3x^9}{24}$$

Outer integral

$$\int_0^1 \frac{6x^7+8x^8+3x^9}{24}\, dx = \frac{1}{24}\left(\frac{3x^8}{4}+\frac{8x^9}{9}+\frac{3x^{10}}{10}\right)\Bigg|_0^1 = \frac{349}{4320}$$