Homework 10: Logistic map

This homework is due Thursday 4/19/2012 at 2359.

(1) Write a function called `logistic` that accepts a parameter λ, an integer n and an initial value of x_0 and returns an array containing the first n iterates of the logistic map as a $1 \times n$ row vector.

```
function x = logistic(lambda, n, x0)

end
```

(2) Write a function called `cobweb` that takes as arguments the value of λ,the number of iterations n and an initial point x_0, and computes the sequence of "x" and "y" values of the points needed for a cobweb diagram.

Use this function to write a script that produces a plot similar to the one in the notese. Run your script for the values of $\lambda = 1, 2, 3, 4$ and for $n = 100$. What do you notice?

```
function [x, y] = cobweb(lambda, n, x0)

end
```

(3) In this exercise we draw a bifurcation diagram. We noted in the previous experiment that the logistic map can exhibit periodic behavior. Oscillating between different values. Produce a plot that has λ on the horizontal axis for $2.5 \leq \lambda \leq 4$ and has the value of x on the vertical axis. Pick some number of iterates, say T (about 500) that will be discarded, and some number K that will be kept (say $K = 100$). Now do the following:

- Start with $\lambda = 2.5$ run the logistic map and generate $K + T$ iterates.
- Discard the first T of these and keep the rest.
- Now plot the values of the iterates vertically above the corresponding value of λ.
- Increment λ by 0.05 and repeat the above process. At the end of the experiment you should have a plot that has K iterates plotted directly above each value of λ.

You may want to write this as a function `bifurcation` that takes as arguments K, T, the size of the increment and the minimum and maximum values of λ.

(4) **Extra credit** Write a function `logistichist` that takes as an argument the value of λ and the number of iterates n. The function should produce a histogram of the first n iterates for this value of λ. Run your function for $\lambda = 3, 3.5, 3.7, 3.8, 4$. What do you notice?

```
function [x, y] = logistichist(lambda, n)

end
```

(5) **Extra credit** Write a function called `logisticcompare`. This function should take as arguments two values of λ a value for the number of iterates n and an intial point x_0. For the two values of λ, call them λ_1, λ_2 we generate the corresponding sequence of iterates, say $x_{1,1}, \ldots, x_{1,n}$ and $x_{2,1}, \ldots, x_{2,n}$. The function should return the array x and a vector d consisting of the differences $d_i = x_{1,i} - x_{2,i}$.

Use this function to plot d, what do you notice?

```
function [d, x] = logisticcompare(lambda1, lambda2, n, x0)

end
```

(6) **Extra credit** Consider the following dynamical system. Choose a number $0 < \alpha < 1$ and consider the sequence

$$x_{n+1} = x_n + \alpha \mod 1.$$

The notation $\mod 1$ is interpreted as follows. If $x_n + \alpha > 1$, then we set $x_{n+1} = x_n + \alpha - 1$. Otherwise we leave it unchanged.

Write a function that generates the first n iterates of this sequence. Call the function `rotation`.

```
function x = rotation(alpha, n, x0)

end
```

By plotting the first 10,000 iterates of this system for $\alpha = \pi/4, \sqrt{2}/2, 1/2, 1/10$ explain how the system behaves.