Homework 5 for SM 233 Section 4031

Due Thursday 2/27/2012 at 2359

This homework assignment has two parts. One part involves writing code, the second part involves proving some facts about stochastic matrices.

This homework is extensive, so start working on it as soon as possible.

Part 1

Write a function called `fractionoftime`. The function should accept as input a matrix `P`, an integer `N` and a state `s`.

`P` is (potentially) a stochastic matrix, and your program should check this by using `isstochastic`.

`N` will be an integer representing the number of trials.

`s` will also be an integer that gives the index of the starting state (note that you should check that the value of `s` is between 1 and n, where n is the number of states)

The function `fractionoftime` will return two values, the first is an array called `states`. The array `states` will have length `N` and `states(i)` will be the state of the chain at the `i`th step. The second value will be called `f`. The array `f` will have n elements, and `f(i)` will be the fraction of time that the process was in state `i`.

Your function should not print anything to the screen, or display any error messages.

The signature of your function will be

```
function [states, f] = fractionoftime(P, N, s)
    ...
end
```

Remember that the function `rand(m, n)` can be used to produce an $m \times n$ array of random numbers distributed uniformly between 0 and 1. You will probably need to make use of a for loop (or a while loop), and perhaps `break`s and `return`s.

Submit your code in a file named `fractionoftime.m` (Don't add your alfa number or name to this filename). You should also submit any other code that you need for the function to run. Only the function `fractionoftime` will be tested.

Part 2

(1) In this question you will work though a complete description of a Markov chain on two states. The transition matrix is of the form

$$P = \begin{bmatrix} a & 1-a \\ 1-b & b \end{bmatrix},$$

where $0 \leq a, b \leq 1$.

(a) For which values of a, b is the state s_1, repectively s_2, absorbing?

(b) Find, and justify, an explicit formula in terms of a, b, k for

(i) the kth power of P

(ii) the eigenvalues, and corresponding eigenvectors of P

(c) For which values of a, b does the limit $\lim_{k\to\infty} P^k$ exist.

(d) Show that $\lim_{m\to\infty} \frac{1}{m+1} \sum_{k=0}^{m} P^k$ always exists.

(e) For which values of a, b is the associated chain regular? ergodic?