Homework 7 for SM 233 Section 4031

This homework is due Thursday 3/22/2012 at 2359.

(1) Given a region $D \subseteq \mathbb{R}^2$ we define the uniform distribution over D by $P(A) = \text{Area}(A)/\text{Area}(D)$ where A is a (reasonable) subset of D. Note that $P(D) = 1$.

Suppose that we want to generate points uniformly from the subset D, i.e., we want samples X_i such that $P(X_i \in A) = \text{Area}(A)/\text{Area}(D)$. How do we go about doing this?

We present two ways of doing this for the unit disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$. Note that $P(A) = \frac{1}{\pi}\text{Area}(A)$ in this case.

Method 1

The first method is to proceed as in the first section. Generate x_n, y_n uniformly in the the interval $[-1, 1]$, compute $P_n = (x_n, y_n)$ and then retain only the points P_n such that $x_n^2 + y_n^2 \leq 1$. Note that this method wastes a lot of points.

Method 2

The second method is to convert to polar coordinates. Since every point in the disk D can be described by a pair (r, θ) such that $0 \leq \theta < 2\pi$ and $0 \leq r \leq 1$ we can generate points directly inside D by first generating r_n uniformly in $[0, 1]$ and θ_n uniformly in $[0, 2\pi)$ and setting $P_n = (r_n \cos(\theta_n), r_n \sin(\theta_n))$.

Write a function called `rejection` and `randpolar` that implement each of these methods. The signatures are:

```
function p = rejection(n)
%REJECTION(n) generate n points inside a square
% of side length 2 and throw away the
% ones that do not lie in the unit disk
% Return p, the points that lie in the disk
% as a 2 x n matrix
end

function p = randpolar(n)
%REJECTION(n) generate n points inside
% the unit disk directly
% using polar coordinates and return
% p the generated points as a 2 x n matrix
end
```

After you write these functions, plot the resulting points on a set of axes with suitable x and y limits. Decide whether the points look uniformly distributed. Submit your comments in the source of each file. You can also put the pictures you generate in the homework folder.

(2) **Extra credit** [This problem was updated 3/21/2012 to incorporate suggestions made by students] Suppose that instead of the unit disk D you have a region that can be described as the area between two curves of the form $y = f(x)$, $y = g(x)$ and the lines $x = a$ and $x = b$. Write a function called `uniformd` which has the signature

```
function p = uniformd(f, g, a, b, n)
% Generate points uniformly from
% the region between f, g
% return a 2 x n matrix
% the first row of which is the x values
% second row is y
...CODE...
end
```

The function should return an $2 \times n$ array of points drawn from the uniform distribution on D.