Homework 8 for SM 233 Section 4031

This homework is due Thursday 3/29/2012 at 2359.

(1) (a) Write a function called `montecarlo` with the following signature

```
function [area, numsamples, muf, sigmaf] = montecarlo(f, e)
%MONTECARLO(f, e)
% e : is the accuracy
% f : is the polynomial given as an array of coefficients
% area : approximate area obtained from Monte Carlo integration
% numsamples : number of samples required to obtain an
%   accuracy of e
% muf : the value of the mean of f over the inteval [0, 1]
% sigmaf : the value of the standard deviation of f
%   over the interval [0, 1]
...CODE..
end
```

We will assume that the function $f$ is a polynomial given by its coefficients array. Use the in-built MATLAB polynomial functions to compute the exact value of the integral $\mu_f = \int_0^1 f(x)\,dx$ and see how many samples $n$ are required to obtain an accuracy of $e$. That is, `numsamples` is the smallest value of $n$ for such that

$$\left| A_n - \int_0^1 f(x)\,dx \right| < e.$$

The function should compute the variance of the polynomial, the quantity that we denote by $\sigma_f$ in the notes. The value of `area` is the approximate area obtained by averaging `numsamples` samples. Your program should return both `Area` and `numsamples`.

(b) Now consider the polynomial $f(x) = x(x-1)(x-1/2)(x-1/3)(x-1/4)$. Use the function `montecarlo` that you wrote in the previous part to check what the values of `numsamples` are for the following values of `e`: 0.1, 0.01, 0.001, 0.0001. What do you notice about the rate at which `n` grows?

You should submit these results in the comments at the bottom of your function, after the closing `end`.

(2) **Extra credit** The function for this question should be called `montecarloimproved` and has the signature

```
function [Area, numsamples] = montecarloimproved(f, e)
...CODE...
end
```

Modify your function from the previous question so that it accepts a function handle `f` as an argument, instead of the coefficients of a polynomial. In this case the integral cannot be computed explicity. This function will compute `numsamples` as the first value of $n$ for which the difference between consecutive averages $A_n$ and $A_{n+1}$ is less than $e$, i.e.,

$$|A_n - A_{n+1}| < e.$$