SM311O Second Exam
30 Oct 1998

1. Let $\psi = x^2 + 3y^2 - 2xy$ be the stream function of a flow.
   (a) Determine the velocity field associated with ψ.
   (b) Determine the line integral of the velocity field along the straight line that connects the points $(1, 2)$ to $(2, -1)$.
   (c) Determine the circulation of this flow around a circle of radius 1 centered at the origin.
2. Let $\mathbf{v} = \langle y \cos x, 2y + \sin x \rangle$.
   (a) Determine whether this velocity field has a potential ϕ. If yes, find ϕ.
   (b) Determine the circulation of this flow around a circle of radius 1 centered at the origin.
3. Let $\mathbf{v} = \langle y, 0 \rangle$.
   (a) Plot some sample velocity vectors for points located on the y-axis.
   (b) Determine the line integral of $\mathbf{v}$ along the ellipse $x^2 + 3y^2 = 5$ when $x > 0$ and $y > 0$.
4. Let $f(x) = x$. Find the Fourier sine series of f in the interval $(0, 3)$.
5. Consider the heat conduction initial–boundary value problem

$$u_t = 4u_{xx}, \quad u(x, 0) = x, \quad u(0, t) = u(3, t) = 0.$$

   (a) Determine the solution to this problem (you may wish to use the result of your computations in Problem 4).
   (b) b. Using only one term of the series solution in part a), determine how long it takes for the temperature of the middle of the bar to reach half of its initial value.
6. In solving for the normal modes of the laplacian in the rectangular region $D = \{(x, y) \,|\, 0 < x < 2, 0 < y < 3\}$, we have arrived at the general solution

$$\psi(x, y) = (c_1 \sin \lambda x + c_2 \cos \lambda x)(c_3 \sin \sqrt{\lambda - \mu^2} y + c_4 \cos \sqrt{\lambda - \mu^2} y).$$

   Determine the constants c_1, c_2, c_3, c_4, λ, and μ so that ψ is a nontrivial normal mode of the laplcian in D.