SM311O Second Exam
9 March 2001

1. **(10 Points)** Consider the Laplace equation $\Delta u = 0$. Verify whether $u(x,y) = 2\sinh 3x \sin 3y$ is a solution of this equation.

2. **(20 points)** Let $\psi = x^2 - y^2 + xy$ be the stream function of a flow.

   (a) Determine the velocity field associated with ψ.

   (b) Determine the circulation of this flow around a circle of radius 2 centered at the origin.

3. **(20 points)** Let $\mathbf{v} = \langle 2xy, x^2 - 2yz + 2, 1 - y^2 \rangle$.

   (a) Determine whether this velocity field has a potential ϕ. If yes, find ϕ.

   (b) Determine the line integral of this flow along the parabola $y = 1 - x^2$ in the $z = 2$ plane from A to B where $A = (1, 0, 2)$ and $B = (2, -3, 2)$.

4. **(20 points)** Let $f(x) = \begin{cases} x & \text{if } 0 \leq x < \frac{1}{2}, \\ 1 - x & \text{if } \frac{1}{2} \leq x < 1. \end{cases}$

   (a) Find the Fourier sine series of f in the interval $(0, 1)$.

   (b) Use the first nonzero term of the Fourier series and evaluate it at $x = \frac{1}{2}$. How much does this value differ from $f(\frac{1}{2})$?

5. **(30 points)** Consider the initial-boundary value problem for the heat equation

$$u_t = 9u_{xx}, \qquad u(x,0) = f(x), \qquad u(0,t) = u(1,t) = 0,$$

   where f is defined in the previous problem.

   (a) Explain in words what $u(x,t)$ and each term in the above equations represent.

   (b) Start with separation of variables and determine the solution to this problem (you may wish to use the result of your computations in the previous problem).

   (c) Using only one term of the series solution in part (a), determine how long ittakes for the temperature at $x = \frac{1}{2}$ to reach 50% of its original temperature.

********* Bonus **********

(10 points) Consider the differential equation

$$-\Delta u = \lambda u. \tag{1}$$

Let $u(x,y) = \sin ax \sin by$. Determine a relationship between λ, a and b for u to be a solution of (1).