SM311O (Third Exam)
23 APRIL 2001

1. (**20 Points**) Give parametrizations for the following curves or surfaces:

   (a) The straight line connecting $(1, 2, 0)$ to $(-1, 4, 0)$.

   (b) The ellipse $2x^2 + 3y^2 = 6$ and located in the plane $z = 1$.

   (c) The plane passing through the points $(1, 0, 0)$, $(0, -1, 0)$ and $(0, 0, 2)$.

   (d) The surface $z = 1 - x^2 - y^2$ with the circular domain $x^2 + y^2 \leq 4$.

2. (**20 Points**) Compute the flux of the velocity field $\mathbf{v} = x\mathbf{k}$ through the surface S, the first quadrant of the disk of radius 3 in the xy-plane centered at the origin and with $x > 0$ and $y > 0$.

3. (**10 points**) Compute the acceleration of the velocity field $\mathbf{v} = \langle \frac{y}{\sqrt{x+y}}, -\frac{x}{\sqrt{x-y}} \rangle$.

4. (**30 Points**) Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ form a basis of unit vectors at a typical point on Earth, which rotates with angular velocity $\mathbf{\Omega}$. Let $\mathbf{v} = u\mathbf{e}_1 + v\mathbf{e}_2 + w\mathbf{e}_3$ be a typical velocity vector measured at a typical point P on Earth.

   (a) Show that $\mathbf{f} = 2\mathbf{\Omega} \times \mathbf{v}$ measures the Coriolis force imparted on the fluid particle P.

   (b) Find the components of $\mathbf{f}$ in terms of Ω, the magnitude of $\mathbf{\Omega}$, and ϕ the latitude of P.

   (c) Compute $\mathbf{f}$ when P is at the north pole; When P is at the equator.

5. (**20 Points**) A flow is called geostrophic if the velocity field $\mathbf{v}$ and pressure gradient ∇p satisfy

$$\rho f v_2 = \frac{\partial p}{\partial x}, \qquad \rho f v_1 = -\frac{\partial p}{\partial y},$$

   where $f = 2\Omega \sin\phi$. Assume the velocity field is incompressible. Let ψ be the stream function of this flow. Prove that the isobars of the flow and particle paths of the flow must coincide. (Hint: Use your knowledge of gradient of a function and its relation to the level curves of that function).