SM311O (Third Exam)

24 April 1998

1. Use double or triple integration to determine the volume of the tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,3,0)$, and $(0,0,4)$.

2. Compute the flux of the velocity field $\mathbf{v} = (1-x^2)\mathbf{j}$ through the surface S, the disk of radius 3 in the xz-plane centered at the origin.

3. Compute the flux of the velocity field $\langle 0,0,z\rangle$ through the surface of the hemisphere $x^2+y^2+z^2<1$ and $z \geq 0$. (Hint: You may calculate the flux directly, or indirectly by applying the divergence theorem.)

4. Let $\mathbf{v} = \langle 2xy - z^2, x^2 - 2yz, -y^2 - 2zx\rangle$.

   (a) Does $\mathbf{v}$ have a potential ϕ? If no, explain. If yes, find it.

   (b) Compute $\int_C \mathbf{v}\cdot d\mathbf{r}$ where C is the straight line connecting $(0,0,0)$ with $(1,1,1)$.

5. Let $\mathbf{v}(x,y,z) = \langle ax^2 - by^2, cxy, 0\rangle$ be the velocity field of a fluid whose density is one and its viscosity is zero. Here a, b, and c are arbitrary real numbers. Find all values of these constants so that the pair $(\mathbf{v}, p)$ satisfies the Navier-Stokes equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \nabla\mathbf{v}\cdot\mathbf{v}\right) = -\nabla p + \mu\Delta\mathbf{v}, \qquad \text{div } \mathbf{v} = 0.$$