SM311O (Third Exam)

21 April 1999

1. (**20 Points**) Give a parametrization of the following surfaces:

   (a) The plane that passes though the points $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 1, 1)$.

   (b) The disk of radius 2 located in the yz-plane and centered at $(0, 2, 3)$.

2. (**15 Points**) Use double integration to determine the volume of the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$.

3. (**15 Points**) Compute the flux of the velocity field $\mathbf{v} = (y - y^2)\mathbf{k}$ through the surface S, the disk of radius 3 in the xy-plane centered at the origin.

4. (**20 Points**) Compute the flux of the vorticity for the velocity field $\mathbf{v} = \langle y, -x, z\rangle$ through the surface of the hemisphere $x^2 + y^2 + z^2 < 1$ and $z \geq 0$. (Hint: You may calculate the flux directly, or indirectly by applying the Stokes theorem, which states that the flux of vorticity along a surface S is equal to the flow's circulation around the boundary of S.)

5. (**30 Points**) Let $\mathbf{v} = \langle \frac{y}{x^2+y^2}, -\frac{x}{x^2+y^2}, 0\rangle$.

   (a) Determine the acceleration of $\mathbf{v}$.

   (b) Determine if the above acceleration has a potential. If it does, find it.