SM311O (Third Exam)
28 APRIL 2000

1. (**20 Points**) Give parametrizations for the following curves or surfaces:

   (a) The straight line connecting $(1,2,3)$ to $(1,4,9)$.

   (b) The disk of radius 2 centered at the origin and located in the plane $z=1$.

   (c) The circle of radius 2, centered at $(1,1,1)$ and located in the plane $z=1$.

   (d) The surface $z=1-x^2-y^2$ with the circular domain $x^2+y^2\leq 1$.

2. (**20 Points**) Compute the flux of the velocity field $\mathbf{v}=x^2\mathbf{k}$ through the surface S, the disk of radius 3 in the xy-plane centered at the origin.

3. (**20 points**) Compute the acceleration of the velocity field $\mathbf{v}=\langle\frac{y}{\sqrt{x^2+y^2}},-\frac{x}{\sqrt{x^2+y^2}}\rangle$.

4. (**15 Points**) Let $\mathbf{\Omega}$ stand for the angular velocity of Earth.

   (a) Assuming that $\mathbf{\Omega}=\Omega\mathbf{k}$ where $\mathbf{k}$ is a unit vector in the z direction, find Ω. (Hint: It takes 24 hours for our planet to go through one rotation.)

   (b) Let $\mathbf{r}$ be the position vector of a typical point P on Earth. We have seen in class that $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$ is the centripetal acceleration at P. Assuming that the radius of Earth is 6000 kilometers, compute the magnitude of the centripetal acceleration at a typical point on the equator.

5. (**15 Points**) Let $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ form a basis of unit vectors at a typical point on Earth, which rotates with angular velocity $\mathbf{\Omega}$. Let $\mathbf{v}=u\mathbf{e}_1+v\mathbf{e}_2+w\mathbf{e}_3$ be a typical velocity vector measured at a typical point P on Earth. We have seen in class that $\mathbf{f}=2\mathbf{\Omega}\times\mathbf{v}$ measures the Coriolis force imparted on the fluid particle P.

   (a) Find the components of $\mathbf{f}$ in terms of Ω, the magnitude of $\mathbf{\Omega}$, and ϕ the latitude of P.

   (b) Compute $\mathbf{f}$ when P is at the north pole; When P is at the equator.

6. (**10 Points**) In the process of studying the problem of the fluid flow induced by an oscillating plate, we came across the differential equation

$$y''-iy=0$$

   where $i=\sqrt{-1}$. Find the general solution of this differntial equation. (Hint: $\sqrt{i}=\pm\frac{1}{\sqrt{2}}(1+i)$.)