Solutions to the Second Homework Set (page 22)

1. (a) **Parametrization**: Let $\mathbf{r}(t) = \langle \cos t, \sin t\rangle$ and $t \in (0, 2\pi)$. The t value for the point $P = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ can be found from the parametrization: We need to find t so that $\mathbf{r}(t) = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$ so t must satisfy the two equations

$$\cos t = \frac{1}{\sqrt{2}}, \qquad \sin t = \frac{1}{\sqrt{2}}$$

from which we conclude that $t = \frac{\pi}{4}$.

Tangent vector: $\mathbf{r}'(t) = \langle -\sin t, \cos t\rangle$. Evaluating $\mathbf{r}'$ at $t = \frac{\pi}{4}$ yields

$$\mathbf{r}'(\frac{\pi}{4}) = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$$

for the tangent vector.

(b) **Parametrization**: Let $\mathbf{r}(t) = \langle 1+\cos t, -1+\sin t\rangle$, $t \in (0, 2\pi)$. Find t so that $\mathbf{r}(t) = \langle 1 + \frac{1}{\sqrt{2}}, -1 + \frac{1}{\sqrt{2}}\rangle$. So

$$1 + \cos t = 1 + \frac{1}{\sqrt{2}}, \qquad -1 + \sin t = -1 + \frac{1}{\sqrt{2}}$$

. So $t = \frac{\pi}{4}$.

Tangent vector: $\mathbf{r}'(t) = \langle -\sin t, \cos t\rangle$ and $\mathbf{r}'(\frac{\pi}{4}) = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$.

(c) **Parametrization**: First divide the equation by 6 to get $\frac{x^2}{2} + \frac{2y^2}{3} = 1$. Now, let $\frac{x^2}{2} = \cos^2 t$ and $\frac{2y^2}{3} = \sin^2 t$, from which we get

$$x(t) = \sqrt{2}\cos t, \qquad y(t) = \sqrt{\frac{3}{2}}\sin t,$$

and, therefore,

$$\mathbf{r}(t) = \langle \sqrt{2}\cos t, \sqrt{\frac{3}{2}}\sin t\rangle.$$

Next, we find t so that $\mathbf{r}(t) = \langle \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\rangle$ or

$$\sqrt{2}\cos t = \frac{1}{\sqrt{2}}, \qquad \sqrt{\frac{3}{2}}\sin t = \frac{\sqrt{3}}{2\sqrt{2}}.$$

Hence, t satisfies

$$\cos t = \frac{1}{2}, \qquad \sin t = \frac{\sqrt{3}}{2},$$

which implies that $t = \frac{\pi}{3}$.

Tangent vector: $\mathbf{r}'(t) = \langle -\sqrt{2}\sin t, \sqrt{\frac{3}{2}}\cos t\rangle$ which when evaluated at $t = \frac{\pi}{3}$ yields $\mathbf{r}'(\frac{\pi}{3}) = \langle -\sqrt{\frac{3}{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\rangle$.

(d) **Parametrization**: