Solutions to the Homework Set on page 93

1. $\mathbf{v} = \langle \frac{y}{\sqrt{x^2+y^2}}, -\frac{x}{\sqrt{x^2+y^2}} \rangle$. The curve C is a circle of radius a centered at $(1,2)$. SInce teh equation of this curve is $(x-1)^2 + (y-2)^2 = a^2$, we parametrize this curve as

$$\mathbf{r}(t) = \langle 1 + a\cos t, 2 + a\sin t \langle, \qquad t \in [0, 2\pi).$$

Note that the velocity field $\mathbf{v}_C$ takes the form

$$\mathbf{v}_C = \langle \frac{2 + a\sin t}{\sqrt{(1+a\cos t)^2 + (2+a\sin t)^2}}, -\frac{1 + a\cos t}{\sqrt{(1+a\cos t)^2 + (2+a\sin t)^2}} \rangle,$$

and that the tangent vector to C has the form

$$\frac{d\mathbf{r}}{dt} = a\langle -\cos t, \sin t \rangle.$$

Hence, the circulation of the velocity field about C, $\oint_C \mathbf{v} \cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{v}_C \cdot \frac{d\mathbf{r}}{dt}\, dt$, becomes

$$f(a) = -\int_0^{2\pi} \frac{a^2 + 2a\sin t + a\cos t}{\sqrt{(1+a\cos t)^2 + (2+a\sin t)^2}}\, dt.$$

The *Mathematica* program in the text produces the graph on page 92.