USNA Summer Seminar 2008

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Summer Seminar ’08

- **Game Theory**
  - Applications to social science, warfare, business, biology, politics, etc.
  - 80-90 minutes
- **Questions** about USNA (esp. academics)
  - 30 minutes

Will Traves

- Associate Professor in the Math Dept.
- At USNA since 1999
- Senior Math Advisor
- Assistant Coach for Women's Rugby Team

Tell us a little about yourself!

- Name
- Hometown
- Fall math class
- Other fun information:
  - Sports teams
  - Extracurricular activities
  - Some other fun fact about you!

Game Theory

- Study of conflict and cooperation
- Two or more players, each having several possible strategies
- The outcome of the game depends on the strategies
- Outcome leads to numerical payoffs to the players
- Example: Rock-Paper-Scissors

A two person zero sum game

Colin

Rose		A	B	C	D
	A	12	-1	1	0
	B	5	1	7	20
	C	3	2	4	3
	D	-16	0	0	16

Play 10 times as practice, then play 10 times, recording strategies used and total score.

Domination

- Is Colin C a reasonable strategy?

		Colin A	Colin B	Colin C	Colin D
Rose	A	12	-1	1	0
Rose	B	5	1	7	20
Rose	C	3	2	4	3
Rose	D	-16	0	0	16

Best Play

- Rose C & Colin B with payoff 2
- saddle point (equilibrium)

Colin

		A	B	C	D
	A	12	-1	1	0
	B	5	1	7	20
Rose	C	3	2	4	3
	D	-16	0	0	16

Domination & Pruning

- Games with players alternating (checkers, chess, etc)
  - Game trees list positions
  - Can prune the game tree to determine the value of the game
- How chess computers play (search & evaluation)
- Checkers is a draw.

Is there always a best strategy?

		Colin		
		A	B	C
Rose	A	2	-3	2
	B	-3	4	-3
	C	2	-3	6

Play game 10 times and record strategies and total score.

Domination

Colin

		A	B	C
	A	2	-3	2
Rose	B	-3	4	-3
	C	2	-3	6

Who has the advantage?
Is there a best single strategy?

Mixed Strategies

Colin

		A	B
Rose	B	-3	4
	C	2	-3

Since opponent can take advantage of any predictable strategy, the only way forward is to use a random strategy!

Expected Payoffs

		Colin	
		A	B
Rose	B	-3	4
	C	2	-3

E(Rose B) = 0.5(-3)+0.5(4) = 0.5
E(Rose C) = 0.5(2) + 0.5(-3) = -0.5

If Colin plays 0.5A, 0.5B then Rose expects to win 0.5 by playing B and lose 0.5 with C.

Rose profits by exploiting the difference in the expected payoffs.

No profit!

Rose profits by exploiting the difference in the expected payoffs. So Colin sets random strategy at xA, (1-x)B so that Rose cannot profit.

Colin

		A	B
Rose	B	-3	4
	C	2	-3

E(Rose B) = x(-3)+(1-x)(4) = 4-7x
E(Rose C) = x(2) + (1-x)(-3) = -3+5x

4-7x = -3+5x
7 = 12x
x = 7/12 so E(Rose B or C) = -1/12

Colin can ensure Rose wins no more than -1/12 on average by playing 7/12A, 5/12B.

Two can play at that game

Rose can also randomize to eliminate the difference in Colin’s expected payoffs.
Rose: yB, (1-y)C

		Colin A	Colin B
Rose	B	-3	4
	C	2	-3

E(Colin A) = y(-3)+(1-y)(2) = -5y+2
E(Colin B) = y(4) + (1-y)(-3) = 7y-3

-5y+2 = 7y-3
5 = 12y
y = 5/12 so E(Colin A or B) = -1/12

Rose can ensure Colin wins no more than 1/12 on average by playing 5/12B, 7/12C.

Summary

- Rose has a mixed strategy so that Colin wins no more than 1/12
- Colin has a mixed strategy so that Colin wins at least 1/12
- If Rose lost more than 1/12 then she could have played better; if Colin wins less than 1/12 then he could have played better
- Value of the game is 1/12 to Colin (i.e. to make the game fair, he should pay Rose 1/12 to play)

Von Neumann's Minimax Theorem

- Every two person zero sum game admits optimal (mixed) strategies for each player.

  The corresponding outcome is the value of the game.

  There are efficient ways to find the best strategies (linear programming).

The Prisoner's Dilemma

- This is an important non-zero sum game

	Squeal	Silent
Squeal	(-1, -1)	(1, -2)
Silent	(-2, 1)	(0, 0)

Both have incentives to squeal, but doing so causes a non-optimal result.

Other applications

- The Prisoner’s dilemma arises in many situations
  - Arms control: both sides have an incentive to cheat, but doing so puts them both in worse positions (RAND Corp.)
  - Business competition: Two nearby gas stations might both have incentive to lower prices (to attract customers); if they both do so then they both lose profits (Nobel prizes)

Other important games

- Games against nature
  - Is nature a rational decision maker?
  - Statistical decision theory (business school)
- Multiplayer games
  - No satisfactory theory about best play
  - General approach is to study formation of coalitions (obvious applications to politics)