Wrestling with Byes at the EIWA Tournament: A Discrete Mathematical Analysis of a Sports Problem

Overview, Summary, and Consequences[1]

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Overview

The Eastern Intercollegiate Wrestling Association (EIWA) holds a championship tournament (the "Easterns") every year. About 13 colleges enter one wrestler in each of ten weight classes. In each weight class several byes must occur in the first round, and these are distributed among the top eight (seeded) wrestlers. The method currently in use distributes the byes at random among the eight seeds. This approach is fast, easy to implement, and perfectly fair to the individual athletes. However, since the random selections for byes are performed in each weight class independently, by sheer chance it sometimes happens that one team receives many more byes than another team, even though both teams have the same number of seeded wrestlers. This situation arose in 2004, when one Ivy League school had 9 seeds and received 0 byes, while a rival had 10 seeds and 7 byes. The two teams thus began the tournament under very different conditions—with a potential impact on the final team placings.

We show that a modification to the current scheme greatly reduces the chances of such an unbalanced situation occurring—while remaining fast, easy to implement, and fair to the individuals. The idea is to generate several complete draws, and then choose the "best" one. We use a precise mathematical notion (least squares) to measure how good a draw is.

[1] A long version of this report includes many more details

[2] This report summarizes the results of the author's own investigations; no endorsement by the Navy wrestling program or coaches is implied or intended.

Summary

We propose a new method, BARRELS-(3), for selecting byes at the Eastern Intercollegiate Wrestling Association championship tournament. (BARRELS is an acronym whose meaning will be explained later.) The BARRELS-(3) method is specified below, and the anticipated consequences are sketched. A careful justification of the method, a discussion of the consequences, and computer-simulated "backtesting" using data from recent tournaments appear in the longer version of this report.

BARRELS-(3) Method[3]

We assume that the coaches have already selected the 8 seeds in each of the 10 weight classes, and that the number of byes in each weight class is known.

1: Find a random draw that distributes byes randomly and independently for all 10 weight classes in the usual manner.[4]

2: Compute the "inequitability penalty to teams" for the random draw.

3: Carry out Steps 1 and 2 three times to generate three complete random draws and their penalties.

4: Select the complete random draw with the lowest penalty.[5]

The inequitability penalty is a single number that measures the difference between the number of byes the teams *expect* to receive in a random draw and the number of byes the teams *actually* receive in a given draw. Smaller penalties correspond to draws that treat the teams more equitably (relative to the number of seeds)

[3] In the event of hardware or software failure, Steps 1 and 2 can be carried out manually. The random draws may be chosen by selecting pieces of paper from a hat, for instance, and the penalty for each draw is easy to compute by hand. (Only addition, subtraction, multiplication, division, and counting are involved.) Of course, a computer is preferred for both making the random selections and computing the penalty.

[4] In other words, select a draw by the method currently in use.

[5] In the unlikely event of a tie for the lowest penalty among several draws, select the draw generated later.

Consequences

• *Draws that assign byes inequitably among the teams (relative to their number of seeds) will occur much less frequently using the BARRELS-(3) method than under the current method.*

For instance, by one measure, a draw as disproportionate (or worse) than the one in 2004 can be expected about once every 29 years under the current method, but only once every 24,000 years under the proposed method.

• *Draws that assign byes very equitably to teams (relative to their number of seeds) will occur much more frequently than under the current method.*

For instance, a draw as equitable as the one in 2002 can be expected only once every 40 years under the current method, but will occur three times as often using the BARRELS-(3) method of draw.

• *Large gaps in the bye distribution among top teams occur less frequently.*

Under the current method of selecting byes, in years when *two* similarly situated teams are contending for the team title (e.g., Cornell and Lehigh in 2005), there is a greater than 25% chance that one team receives *at least three more byes* than the other.

Under the proposed BARRELS-(3) method, that chance drops to less than 16%.

When *three* similarly situated teams are contending for the team title (e.g., Cornell, Lehigh, and Penn in 2004), one team receives *at least four more byes* than another about once every 4 years under the current method.

Under the proposed BARRELS-(3) method, that disparate occurrence drops to less than once every 10 years.

• *A seeded wrestler in a weight class with 4 byes has a 50% chance exactly of receiving a bye under the current method. In the proposed method the probability of a bye falls within a narrow range centered at 50%. The probabilities fall between 49.5% and 50.5%. (Similar assertions hold for weight classes with other than 4 byes.)*

These tiny deviations in individual bye probabilities are the only geniune drawback to the proposed BARRELS-(3) method. Someone's decision to support or reject the BARRELS-(3) method should be based on his answer to the following question: The BARRELS-(3) method is substantially more likely to produce draws that treat the teams more equitably (relative to number of seeds); are the tiny deviations in individual bye probabilities an acceptable price for the equitable treatment of teams?

In most cases the individual bye probabilities are actually restricted to a narrower range (about 49.75% to 50.25%). Moreover, it is extremely difficult to predict or determine each wrestler's probability of a bye within this narrow range; no individual can reasonably claim his probability of a bye differs from 50% by an amount he can quantify. Even determining whether an individual's bye probability increases or decreases from 50% is difficult.

• **In a nutshell:** There is a trade-off between perfect fairness to individual wrestlers and an equitable treatment of the teams with respect to distribution of the byes at the EIWA tournament. The current method is perfectly fair to individuals, but can treat the teams inequitably or very inequitably. The proposed method gives up a very tiny amount of fairness to individuals and achieves a far more equitable treatment of teams. The deviation from perfect fairness to individuals is so small that it is difficult to measure even with a computer. Theoretically, it would take tens of thousands of years for the lack of perfect fairness to individual wrestlers to be noticed. In practice, an individual wrestler is far more likely to be treated unfairly by other matters inherent in conducting a tournament. For instance, the coaches are unable to seed the wrestlers with perfect accuracy in each weight class. Also, the clocks are not stopped and started with complete accuracy during matches.