MATHEMATICS AND COLLEGIATE WRESTLING:

WRESTLING WITH BYES

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Summary

An annual, major collegiate wrestling tournament has been improved using mathematics.

We use the familiar least squares method 'in reverse' to distribute the byes more equitably among the teams, while preserving fairness to individual wrestlers.

For More Information

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website includes FAQ

EIWA

Eastern Intercollegiate Wrestling Association

- ◇ 13 teams
- ◇ 10 weight classes

League Championship Tournament

- ◇ qualifier for NCAA meet
- ◇ 13 wrestlers in each weight class
- ◇ standard 16-person bracket

Uh-Oh! $13 = 16 - 3$

We need 3 byes in the first round.
This happens in each of the 10 weight classes.

Question

How do we allocate the three byes?

Answer (Axiom)

Distribute 3 byes randomly among the 8 seeded wrestlers in each weight class.

Consequence

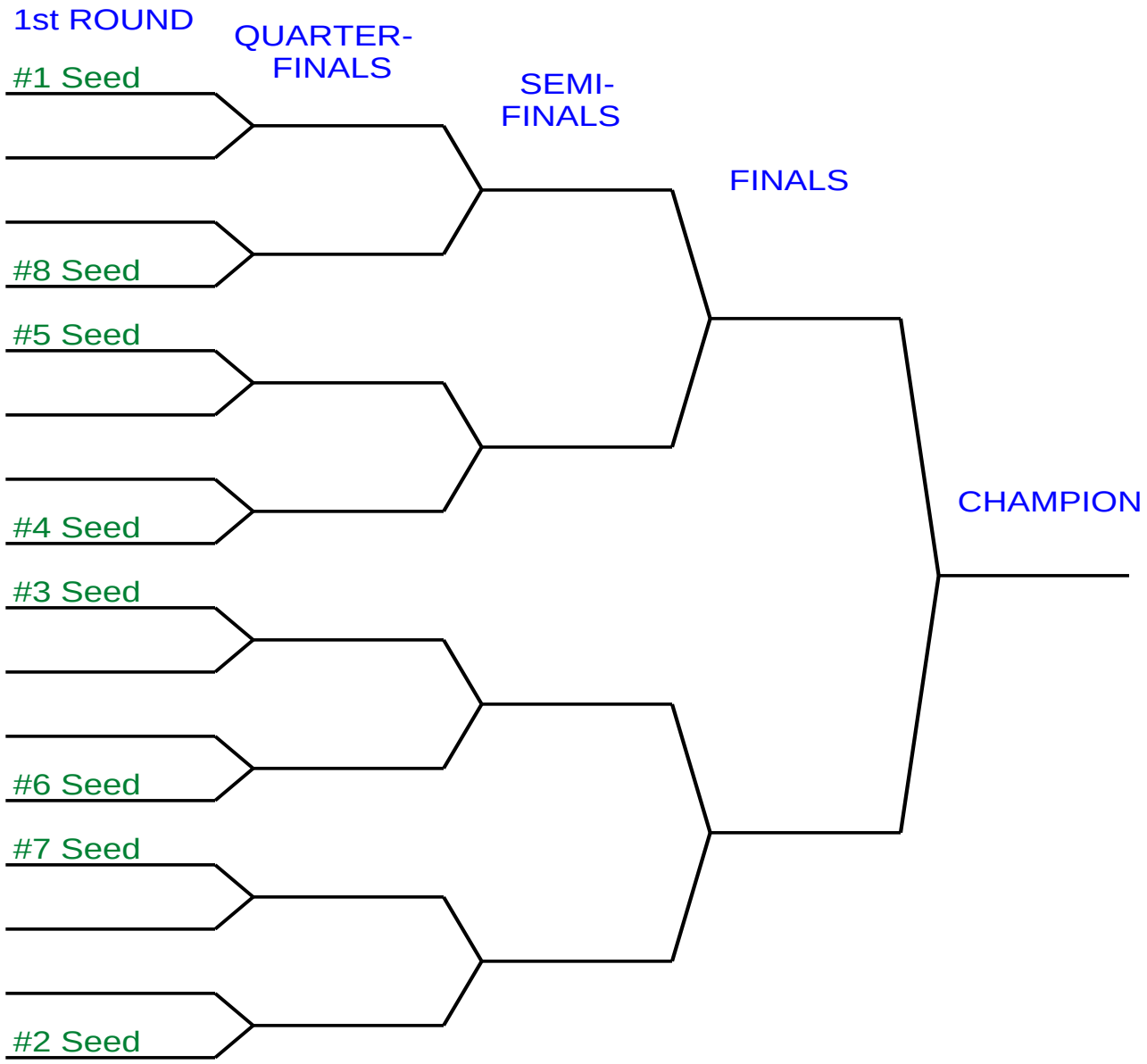
Each of the 8 seeded wrestlers has a

$$37.5\% \quad (=3\text{-out-of-}8)$$

chance of getting a bye.

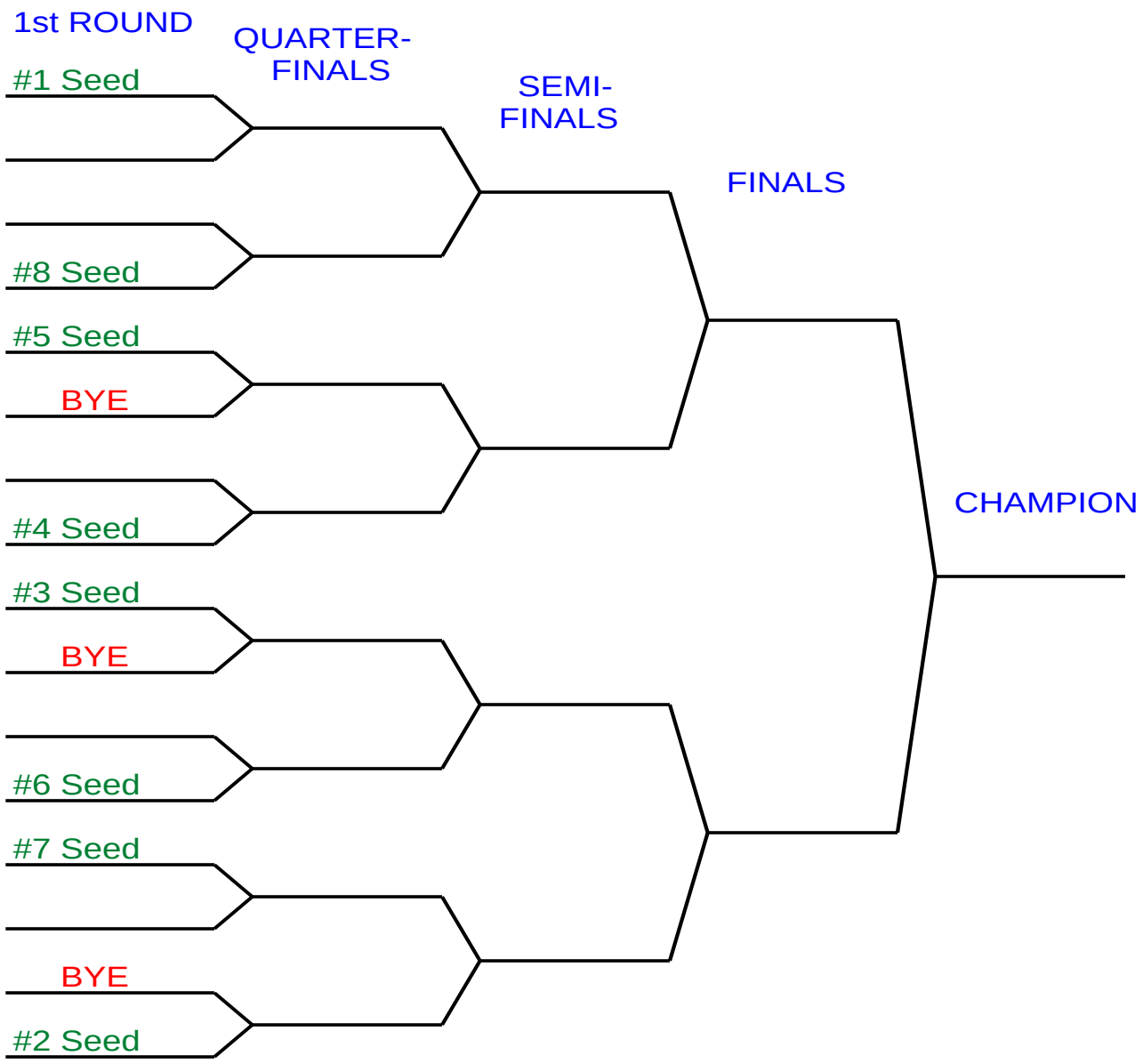
How to Make an EIWA Wrestling Bracket

1. Place the 8 seeded wrestlers far apart in bracket



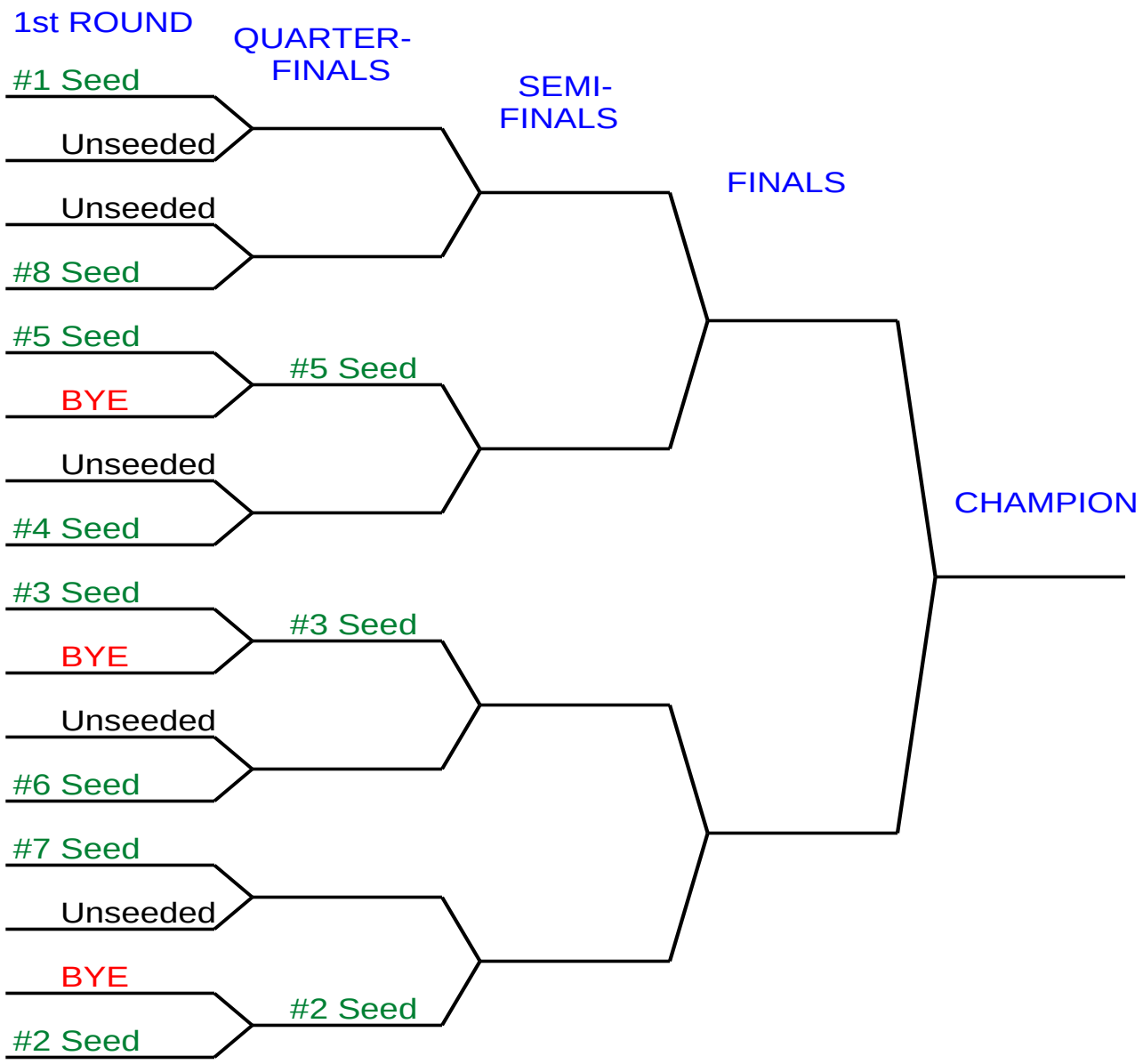
How to Make an EIWA Wrestling Bracket

1. Place the 8 seeded wrestlers far apart in bracket
2. Distribute 3 first-round byes at random among seeds



How to Make an EIWA Wrestling Bracket

1. Place the 8 seeded wrestlers far apart in bracket
2. Distribute 3 first-round byes at random among seeds
3. Randomly pair 5 remaining seeds w/ unseeded wrestlers



Byes and Seeds for 2002

byes received	byes expected	team	seeds
3	3.625	Cornell	9
4	4.000	Penn	10
2	3.625	Lehigh	9
2	2.750	Rutgers	7
3	2.750	Navy	7
3	2.750	Army	7
4	3.250	Harvard	8
4	1.625	Columbia	4
3	2.000	ESU	5
3	2.750	Brown	7
0	0.000	F&M	0
3	2.125	Princeton	5
1	0.750	Bucknell	2
35 total	35.000	EIWA	80

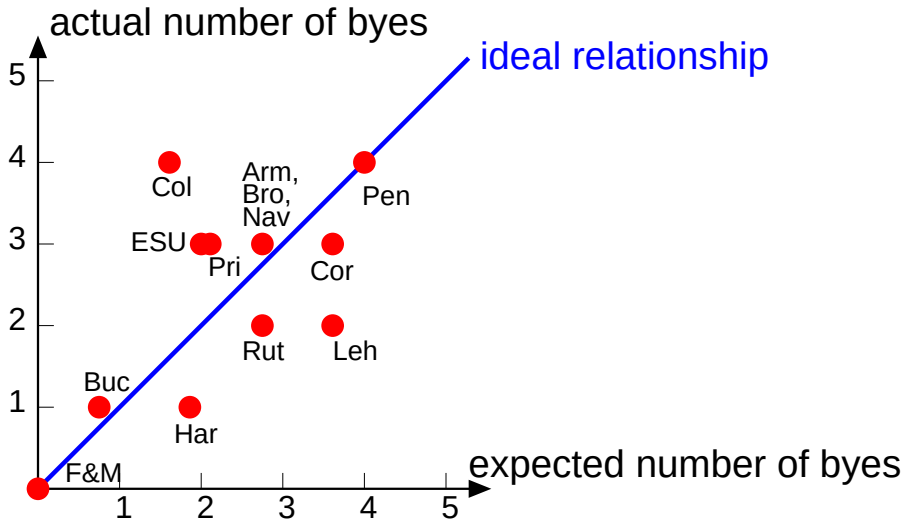
What Happened in 2004?

byes received	byes expected	team	seeds
7	4.125	Cornell	10
0	3.750	Penn	9
3	3.750	Lehigh	9
3	3.750	Rutgers	9
3	3.750	Navy	9
5	3.250	Army	8
1	1.825	Harvard	5
4	2.500	Columbia	6
4	1.750	ESU	4
1	1.250	Brown	3
2	1.750	F&M	4
0	0.750	Princeton	2
0	0.750	American	2
33 total	33.000	EIWA	80

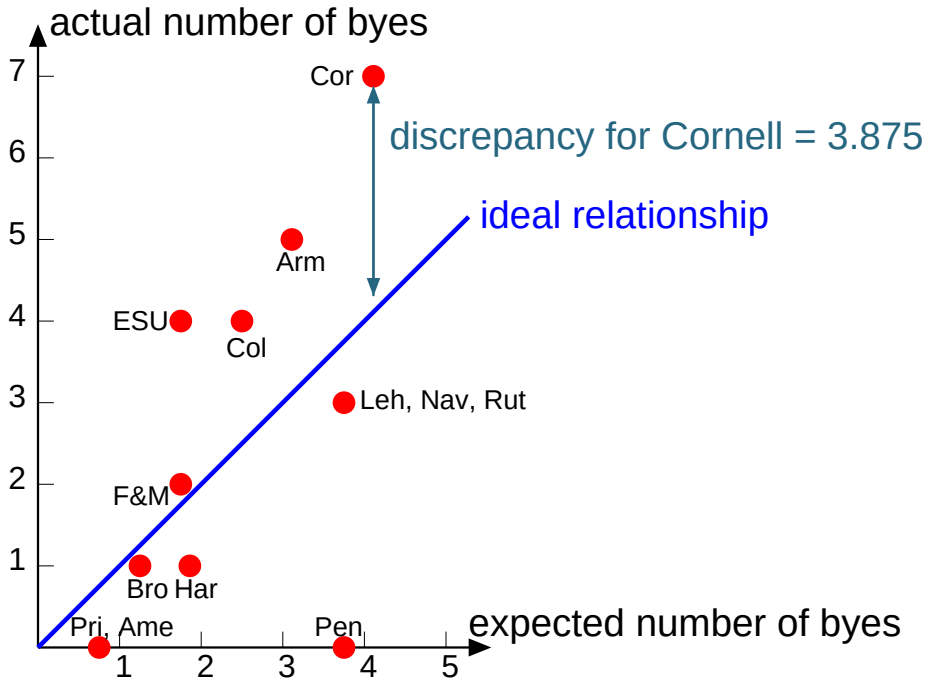
The disparity in the byes among the top three teams could change the team-title race through first-round bonus points.

Graphic Display of Data

2002



2004



The Penalty of a Draw

Suppose we have produced a random draw—the brackets for all 10 weight classes.

Each team has a discrepancy:

discrepancy = byes received − byes expected.

The penalty of a draw is the sum of the squares of the discrepancies:

$$\text{penalty} = \sum_{\text{teams}} (\text{discrepancy})^2$$

The penalty measures how disproportionately the byes are distributed among all the teams.

large penalty $\Longleftrightarrow$ draw distributes byes poorly.
small penalty $\Longleftrightarrow$ draw distributes byes well.

New Method (2006-present)

1. Make a complete random draw.

2. Compute the penalty of the draw.

3. If the penalty exceeds a cut-off,

   then reject the draw.

   Go back to Step 1.

4. If the penalty does not exceed the cut-off,

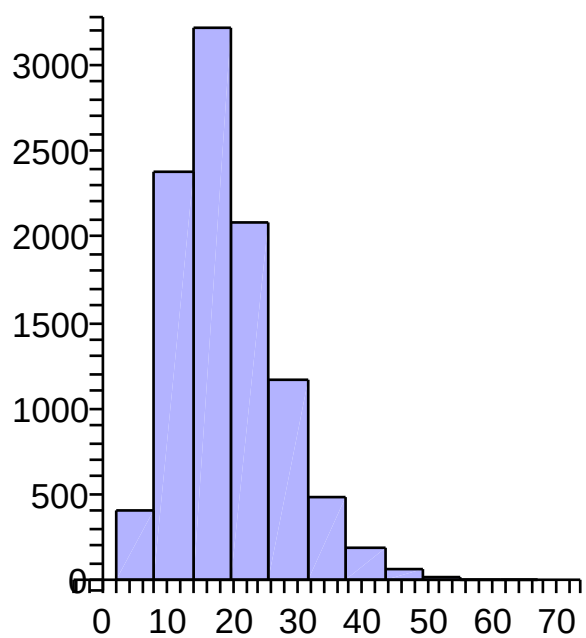
   then accept the draw. Begin wrestling!

The cut-off is chosen in advance....
... by a method explained later.

Penalty Distributions for a Random Draw

large penalty $\Longleftrightarrow$ draw distributes byes poorly.
small penalty $\Longleftrightarrow$ draw distributes byes well.

year	penalty
2002	6.6
2003	12.0
2004	36.4
2005	20.8



Simulation: $n = 10,000$; median $= 17.9$; mean $= 19.0$

First Draw for 2006

byes received	byes expected	team	seeds
1	4.125	Lehigh	10
5	4.125	Navy	10
6	3.750	Penn	9
5	3.750	Cornell	9
7	3.625	Army	9
1	2.875	American	7
1	2.125	Harvard	5
1	2.125	Columbia	5
3	2.125	ESU	5
2	2.000	Brown	5
0	1.625	Rutgers	4
0	0.375	F&M	1
1	0.375	Princeton	1
33 total	33.000	EIWA	80

Penalty: $38.53 > \underbrace{17.19}_{\text{cut-off}}$ Reject this draw!

Fifth (and Final) Draw for 2006

byes received	byes expected	team	seeds
5	4.125	Lehigh	10
4	4.125	Navy	10
4	3.75	Penn	9
3	3.75	Cornell	9
3	3.625	Army	9
5	2.875	American	7
3	2.125	Harvard	5
1	2.125	Columbia	5
2	2.125	ESU	5
2	2.000	Brown	5
1	1.625	Rutgers	4
0	0.375	F&M	1
0	0.375	Princeton	1
33 total	33.000	EIWA	80

Penalty: $9.00 \leq \underbrace{17.19}_{\text{cut-off}}$ Accept this draw!

We have produced a draw that distributes byes reasonably and remains fair to individuals.

The Penalty Cut-Off Number (Extreme Cases)

cut-off $= \infty$: accept first draw. Old method.

- ◇ teams: unfair some years
- ◇ wrestlers: perfectly fair

cut-off ≈ 0: 'quota method'

- ◇ teams: very fair
- ◇ wrestlers: unfair to some unacceptable!

The quota method:

- basically assigns the number of byes to each team
- skews bye-probabilities for seeds away from 37.5%
- is therefore unacceptable

How to Choose the Penalty Cut-Off Number

We use an intermediate value for the cut-off that greatly reduces the unfairness to teams, while preserving fairness to individual wrestlers.

The cut-off is chosen so that each seeded wrestler's bye-probability is skewed by less than 0.01

Extensive simulations were required to determine the cut-off for each scenario—total number of byes, total number of teams.

The simulations used actual seed data from recent years as well as artificial test-case scenarios.