SM 223$_{\text{TSM}}$ Quiz #12 2 Nov 2010 Last Name SOLUTIONS Section ○4041 ○5041 ○6041

Time Limit: 5 minutes

Instructions: Open book. Closed notes. No calculator allowed.

Instructions for all quizzes: **Do not discuss any aspect of this quiz with other midshipmen until after 6th period.**

Print your last name above. Also, fill in the bubble for your section.

Fill the bubble for the correct answer. Also, write your answers in any blanks provided.

Your work will not be graded unless the instructions request you show your work.

For this quiz we consider the point $P = (4, -1)$ and the function

$$f(x, y) = 3y\sqrt{2x + 1}.$$

1. Find the gradient of f at P.

   ● $\langle -1, 9\rangle$ ○ $\langle -1, 9/2\rangle$ ○ $\langle -1/2, 9\rangle$ ○ $\langle -1/2, 9/2\rangle$ ○ $\langle 1/2, -9/2\rangle$
   ○ none of above; correct is _____

   Reason: $f(x, y) = 3y(2x + 1)^{1/2}$. So

   $$\nabla f = \left\langle f_x, f_y \right\rangle = \left\langle 3y\frac{1}{2}(2x + 1)^{-1/2}\cdot 2, 3(2x+1)^{1/2} \right\rangle = \left\langle \frac{3y}{\sqrt{2x + 1}},\ 3\sqrt{2x + 1} \right\rangle.$$

   Therefore

   $$\nabla f(P) = \nabla f(4, -1) = \left\langle \frac{3y}{\sqrt{2x + 1}},\ 3\sqrt{2x + 1} \right\rangle \Bigg|_{(x,y)=(4,-1)} = \langle -1, 9\rangle$$

2. Find the maximum rate of change of f at P.

   ○ $\sqrt{82}/4$ ○ $\sqrt{85}/2$ ○ $5\sqrt{13}/4$ ● $\sqrt{82}$ ○ $\sqrt{102}$ ○ none of above; correct is _____

   Reason: Maximum rate of change of f at P is

   $$|\nabla f(P)| = |\langle -1, 9\rangle| = \sqrt{(-1)^2 + 9^2} = \sqrt{82}.$$

3. Find the directional derivative of f at P in the direction from P to $Q = (1, 3)$.

   ○ $\sqrt{82}/5$ ○ $\sqrt{39}/5$ ○ $26/5$ ● $39/5$ ○ 26 ○ none of above; correct is _____

   Reason: The vector $\overrightarrow{PQ}$ is $\langle 1 - 4,\ 3 - (-1)\rangle = \langle -3, 4\rangle$, which has length 5. So a unit vector in the same direction as the vector $\overrightarrow{PQ}$ is $\mathbf{u} = \frac{1}{5}\langle -3, 4\rangle$. The desired directional derivative is

   $$D_{\mathbf{u}}f(P) = \nabla f(P) \cdot \mathbf{u} = \langle -1, 9\rangle \cdot \langle -3, 4\rangle\frac{1}{5} = \frac{3 + 36}{5} = \frac{39}{5}.$$