SM 223_{TSM} **Quiz #14 17 Nov 2010 Last Name** SOLUTIONS Section ○ 4041 ○ 5041 ○ 6041

Time Limit: 4 minutes

Instructions: Closed book. Closed notes. No calculator allowed.

Instructions for all quizzes: **Do not discuss any aspect of this quiz with other midshipmen until after 6th period.**

Print your last name above. Also, fill in the bubble for your section.

Fill the bubble for the correct answer. Also, write your answers in any blanks provided.

Your work will not be graded unless the instructions request you show your work.

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1. Let $f(x,y)$ satisfy

$$f_x(a,b) = f_y(a,b) = 0,$$

$$f_{xx}(a,b) = -2 \qquad f_{yy}(a,b) = -5, \qquad f_{xy}(a,b) = 3.$$

How should we classify the point (a,b)?

- ○ not a critical point
- ○ a saddle point
- ○ a relative minimum
- ● a relative maximum
- ○ none of the above

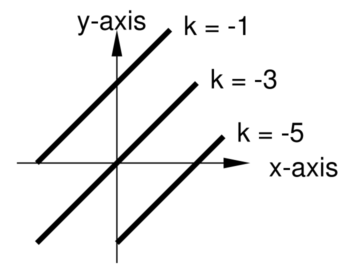
Reason: Since $f_x(a,b) = f_y(a,b) = 0$, the point (a,b) is definitely a critical point. The discriminant of f at (a,b) is

$$D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2 = (-2)(-5) - (3)^2 = 1 > 0.$$

The discriminant is positive, so we have more work to do. We have $f_{xx}(a,b) = -2 < 0$. So (a,b) is a relative maximum by the Second Derivative Test.

2. The contour map shows the level curves $f(x,y) = k$ for $k = -1, -3, -5$. Which vector could be the gradient of f at the origin?

○ $\mathbf{i} + \mathbf{j}$ ○ $2\mathbf{i} + 2\mathbf{j}$ ● $-\mathbf{i} + \mathbf{j}$ ○ $-\mathbf{i} - \mathbf{j}$ ○ $3\mathbf{i} + 3\mathbf{j}$ ○ $\mathbf{i} - \mathbf{j}$



Reason: The gradient vector must be perpendicular to the level curve through the origin, which eliminates all the given vectors except $\mathbf{i} - \mathbf{j}$ and $-\mathbf{i} + \mathbf{j}$. Also, the gradient must point in the direction of maximum rate of increase of f at the origin. From the labels on the level curves, f increases in the direction of Quadrant II at the origin, so the only choice is $-\mathbf{i} + \mathbf{j}$.