SM 223$_{\text{TSM}}$ Quiz #15 19 Nov 2010 Last Name SOLUTIONS Section ○4041 ○5041 ○6041

Time Limit: 5 minutes

Instructions: Closed book. Closed notes. No calculator allowed.

Instructions for all quizzes: **Do not discuss any aspect of this quiz with other midshipmen until after 6th period.**

Print your last name above. Also, fill in the bubble for your section.

Fill the bubble for the correct answer. Also, write your answers in any blanks provided.

Your work will not be graded unless the instructions request you show your work.

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1. Complete the formula for the area of a rectangle with base x and height y. $A = xy$

   The base of a certain rectangle is always increasing at the rate of 2 m/hr, but the area remains constant at 48 m^2.

   At a certain instant the base of the rectangle is 4 m.

2. What is the height of the rectangle at this instant?

   ○ $y = 4$ ○ $y = 6$ ○ $y = 8$ ● $y = 12$ ○ $y = 16$

   Reason. Note that when $x = 4$, we have $48 = A = xy = 4y$, and so $y = 12$.

3. What can we say about how the height of the rectangle is changing at this same instant?

   FILL IN A BUBBLE—AND A BLANK, IF APPROPRIATE.

   - ○ We cannot say anything definitive
   - ○ The height is not changing
   - ○ The height is decreasing at some rate we cannot determine
   - ○ The height is increasing at some rate we cannot determine
   - ● The height is decreasing at the rate 6 m/hr
   - ○ The height is increasing at the rate _____

   Solution: First, observe that since the area is constant, and the base is increasing, the height must be decreasing. This narrows down the list of answers to just two possibilities.

   We know $A(x, y) = xy$, and $\frac{dx}{dt} = 2$, and $\frac{dA}{dt} = 0$. We seek $\frac{dy}{dt}$ when $x = 4$ and $y = 12$. By the Chain Rule (draw your own tree diagram),

   $$\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt}.$$

   Therefore

   $$0 = y\frac{dx}{dt} + x\frac{dy}{dt} = (12)(2) + 4\frac{dy}{dt}.$$

   Algebra gives $\boxed{\frac{dy}{dt} = -6\,\text{m/hr.}}$ So the height is decreasing at a rate of 6 m/hr.