SM 223$_{\text{TSM}}$ Quiz #7 5 Oct 2010 Last Name SOLUTIONS Section ○4041 ○5041 ○6041

Time Limit: 10 minutes

Instructions: Open book. Open notes. Calculator allowed.

Instructions for all quizzes: **Do not discuss any aspect of this quiz with other midshipmen until after 6th period.**

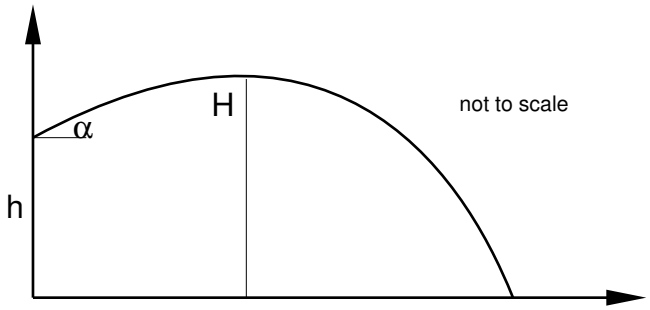
Print your last name above. Also, fill in the bubble for your section.

Fill the bubble for the correct answer. Also, write your answers in any blanks provided.

Your work will not be graded unless requested.

A projectile is fired from a cliff of height h with initial speed v_0 at an angle α above horizontal. The x- and y-axes are in their usual orientations with the origin at the base of the cliff. Distance is measured in meters, and time in seconds. The projectile reaches its maximum height H. This experiment occurs on a planet where the acceleration due to gravity is $\mathbf{g} = \langle 0,\ -g \rangle$. (Do not assume that $g = 9.8$.) The projectile's motion is given by:

$$x(t) = 40t$$
$$y(t) = -5t^2 + 30t + 80$$



1. Fill in the correct bubble for each parameter.
   Give the angle α to the nearest degree. **Hint:** $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{v_0 \sin(\alpha)}{v_0 \cos(\alpha)}$

h	g	v_0	α	H
○ 30 m	○ 5 m/s^2	○ 40 m/s	○ 27°	○ 80 m
○ 40 m	● 10 m/s^2	● 50 m/s	● 37°	○ 100 m
○ 60 m	○ 20 m/s^2	○ 60 m/s	○ 47°	○ 120 m
○ 70 m	○ 30 m/s^2	○ 70 m/s	○ 57°	● 125 m
● 80 m	○ 40 m/s^2	○ 80 m/s	○ 67°	○ 130 m

Our general equations for motion are

$$x(t) = (v_0 \cos(\alpha))\, t \qquad \text{and} \qquad y(t) = -\frac{1}{2}\, gt^2 + (v_0 \sin(\alpha))\, t + h.$$

$\boxed{h:}$ $h = y(0) = 80$

$\boxed{g:}$ Match the quadratic coefficients in the expression for y to see that $-5t^2 = -\frac{1}{2}gt^2$. So $5 = \frac{1}{2}g$ and $g = 10$.
OR: Compute acceleration in the y-direction as $y''(t) = -10$.

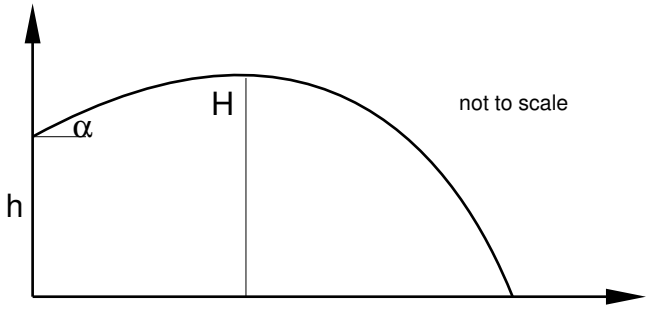
$\boxed{v_0:}$ The velocity vector is $\langle x'(t), y'(t) \rangle = \langle 40,\ -10t + 30 \rangle$.
So the initial velocity vector is $\langle x'(0), y'(0) \rangle = \langle 40,\ 30 \rangle$.
The initial speed is $v_0 = |\langle 40, 30 \rangle| = \sqrt{40^2 + 30^2} = 50$.

$\boxed{\alpha:}$ We have $\tan(\alpha) = \frac{v_0 \sin(\alpha)}{v_0 \cos(\alpha)} = \frac{30}{40}$. So $\alpha = \arctan(\frac{30}{40}) \approx 37°$.

$\boxed{H:}$ The maximum height occurs when $0 = y'(t) = -10t + 30$, i.e, at time $t = 3$. So $H = y(3) = -5(3)^2 + 30(3) + 80 = 125$.

2. Now suppose a radio tower with height 50 meters is located 280 meters down-range. Does the projectile fall short of the tower, hit the tower, or sail over the tower?

$$x = x(t) = 40t$$
$$y = y(t) = -5t^2 + 30t + 80$$



FILL IN A BUBBLE. ALSO INDICATE THE VALUE OF D

- ○ Falls short: The projectile lands D meters in front of the base of tower.
- ● Hits: The projectile hits the tower D meters from the top.
- ○ Sails over: The projectile lands D meters beyond the base of tower.

● $D = 5$ ○ $D = 10$ ○ $D = 20$ ○ $D = 40$ ○ $D = 45$

Reason: The correct range occurs when $280 = x(t) = 40t$, that is, at $t = 7$. At that time we have $y(7) = 45$. So the projectile hits the tower 5 meters from the top of the 50 meter tower.