

Time Limit: 4 minutes

Instructions: Closed book. Closed notes. Calculator allowed.

Instructions for all quizzes: Do not discuss any aspect of this quiz with other midshipmen until after 6th period.

Print your last name above. Also, fill in the bubble for your section.

Fill the bubble for the correct answer. Also, write your answers in any blanks provided.

Your work will not be graded unless requested.

1. Complete the formula for the length of the space curve $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ for $t = a$ to $t = b$:

$$L = \int_a^b |\mathbf{r}'(t)| dt \quad \text{OR} \quad \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$

2. The position of a particle at time t is given by

$$\mathbf{r}(t) = \left\langle 2t, \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 \right\rangle \quad (0 \leq t \leq 6)$$

- (a) Find the speed of the particle at $t = 3$.

1 2 3 4 5 none of above; correct is _____

Reason: The velocity is $\mathbf{r}'(t) = \langle 2, 2t^{1/2}, t \rangle$.

So the velocity at $t = 3$ is $\mathbf{r}'(3) = \langle 2, 2\sqrt{3}, 3 \rangle$.

The speed is $|\mathbf{r}'(3)| = |\langle 2, 2\sqrt{3}, 3 \rangle| = \sqrt{2^2 + (2\sqrt{3})^2 + 3^2} = \sqrt{25} = 5$.

- (b) Find the total distance traveled by the particle for $0 \leq t \leq 6$.

6 8 12 24 30 none of above; correct is _____

Reason: The speed at time t is

$$|\mathbf{r}'(t)| = |\langle 2, 2t^{1/2}, t \rangle| = \sqrt{2^2 + (2\sqrt{t})^2 + t^2} = \sqrt{4 + 4t + t^2} = \sqrt{(2+t)^2} = 2+t.$$

So the distance traveled is

$$L = \int_a^b |\mathbf{r}'(t)| dt = \int_0^6 (2+t) dt = \left(2t + \frac{1}{2}t^2 \right) \Big|_0^6 = 30.$$

Note: Your calculator will also evaluate the integral

$$L = \int_a^b |\mathbf{r}'(t)| dt = \int_0^6 \sqrt{2^2 + (2\sqrt{t})^2 + t^2} dt = \int \text{sqrt}(2^2 + (2\text{sqrt}(t))^2 + t^2), t, 0, 6) = 30$$