SM 223 Test #1 21 Sep 2009 Last Name _____ **Period** ○ 1st ○ 2nd ○ 3rd

Instructions

0. Failure to follow instruction can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are **7** problems altogether. Relative weights are given in the table.
5. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
6. If a problem requests you to show your work, use the space provided to receive credit.
7. **Calculators are not allowed for this test.**
8. Unless otherwise indicated, leave answers in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature _____

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	**Points**	**Score**
1	30	
2	30	
3	30	
4	30	
5	35	
6	35	
7	60	
Total	**250**	

Test Score	%	**Grade**
200	80	A
175	70	B
150	60	C
< 150	< 60	F

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1. This problem deals with the point $P = (1, 2, 12)$ and the plane

$$2x + 2y + z = 36.$$

(a) Give parametric equations for the line through P perpendicular to the plane.

_____

(b) Find the point on the plane that is closest to P.

Put your answer here: ___(, ,)___

Show your work below.

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2. Each equation defines a surface in three dimensions.
Identify each surface by writing the **best** capital letter [A]–[L] in the blank.
A capital letter may be used more than once or not at all.

A:	plane	D:	paraboloid	G:	hyperboloid of 2 sheets	J:	parabolic cylinder
B:	sphere	E:	ellipsoid	H:	hyperbolic paraboloid	K:	two parallel planes
C:	cone	F:	circular cylinder	I:	hyperboloid of 1 sheet	L:	two intersecting planes

____ $z = x^2 + y^2$

____ $z = x^2 - y^2$

____ $x^2 + y^2 = 12$

____ $x^2 + y^2 = 12 - z$

____ $-x^2 + y^2 - z^2 = 12$

____ $z = (4 + y)(4 - y)$

____ $z^2 = 1$

____ $z^2 - x^2 = y^2$

____ $\langle 1, -2, 3\rangle \cdot \langle x, y, z\rangle = 12$

____ $\det \begin{bmatrix} x^2 & y^2 & z^2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = 12$

3. Identify each curve by writing the **best** capital letter (A)–(J) in the blank.
A capital letter may be used more than once or not at all.

A:	circle	F:	line
B:	semi-circle	G:	line segment
C:	quarter-circle	H:	parabola
D:	ellipse	I:	spiral
E:	semi-ellipse	J:	helix

____ $\mathbf{r}(t) = \langle \cos(t),\, \sin(t) \rangle$

____ $\mathbf{r}(t) = \langle t\cos(t),\, t\sin(t) \rangle$

____ $\mathbf{r}(t) = \langle \cos(t),\, \sin^2(t) \rangle$

____ $\mathbf{r}(t) = \langle 3\cos(2t),\, 4\sin(2t) \rangle \quad (0 \le t \le \pi)$

____ $\mathbf{r}(t) = \langle \cos(2t),\, \sin(2t),\, 2t \rangle$

____ $\mathbf{r}(t) = \langle \cos(2t),\, \sin(2t),\, 2 \rangle$

____ $\mathbf{r}(t) = \langle 12\sin(t),\, 12\cos(t) \rangle$

____ $\mathbf{r}(t) = \langle t,\, 2t,\, 3t \rangle \quad (0 \le t \le 12)$

4. TRUE or FALSE.
Fill in a bubble in each row.
All statements deal with three dimensions.

TRUE	FALSE	
○	○	Two lines parallel to a third line must be parallel to each other.
○	○	Two lines perpendicular to a third line must be parallel to each other.
○	○	Two planes parallel to a third plane must be parallel to each other.
○	○	Two lines perpendicular to a plane are parallel to each other.
○	○	Two planes perpendicular to a line are parallel to each other.
○	○	Two lines either intersect or are parallel.
○	○	A plane and a line either intersect or are parallel.

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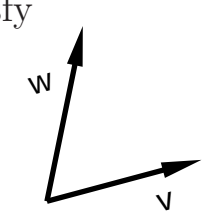
5. The sketch shows two vectors $\mathbf{v}$ and $\mathbf{w}$. The vectors satisfy

$$|\mathbf{v}| = 4 \qquad |\mathbf{w}| = 6 \qquad \mathbf{v} \cdot \mathbf{w} = 8$$



(a) Draw the vector projection of $\mathbf{v}$ onto $\mathbf{w}$ in the sketch at a suitable position.

(b) Find $\mathbf{v} \cdot \mathbf{v}$.

○ 0 ○ 2 ○ 4 ○ 8 ○ 16

(c) Find $\mathbf{w} \cdot \mathbf{v}$.

○ -8 ○ 8 ○ 0 ○ -24 ○ 24

(d) Find the scalar projection of $\mathbf{v}$ along $\mathbf{w}$.

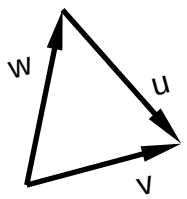
○ 1/3 ○ 1/2 ○ 1 ○ 4/3 ○ 2

(e) The angle θ formed by the two vectors satisfies $\cos(\theta) =$

○ 1/4 ○ 1/3 ○ 1/2 ○ 2/3 ○ 3/4

(f) Which expression is equal to the vector $\mathbf{u}$ in the diagram?

○ $\mathbf{v} + \mathbf{w}$ ○ $\mathbf{v} - \mathbf{w}$ ○ $\mathbf{w} - \mathbf{v}$ ○ $\left(\frac{1}{|\mathbf{v}|}\right)\mathbf{v}$ ○ $\left(\frac{1}{|\mathbf{w}|}\right)\mathbf{w}$



6. Consider the three points

$$P = (1, 3, 3), \qquad Q = (4, 5, 9), \qquad R = (2, 5, 5).$$

Here are some facts you should use to answer the fill-in-the-blank and multiple choice questions:

$$\mathbf{PQ} = \langle 3, 2, 6\rangle, \quad |\mathbf{PQ}| = 7, \quad \mathbf{PQ} \cdot \mathbf{PR} = 19, \quad \mathbf{PQ} \times \mathbf{PR} = \langle -8, 0, 4\rangle.$$

(a) Find a unit vector parallel to **PQ**: $\langle \quad , \quad , \quad \rangle$

(b) Find a vector parallel to **PQ** with length 14: $\langle \quad , \quad , \quad \rangle$

(c) Give an equation of the plane through P, Q, and R:

(d) Find the work done by the constant force **PQ** in moving an object on a straight line from P to R.

Note: The distance is in meters and force is in Newtons. So the work is in Joules.

○ 29 J ○ 19 J ○ 80 J ○ $\sqrt{80}$ J ○ 4 J

(e) Find the area of $\triangle PQR$.

○ $\sqrt{80}$ ○ $\frac{1}{2}\sqrt{80}$ ○ $\sqrt{19}$ ○ $\frac{1}{2}\sqrt{19}$ ○ 12

7. This problem has 8 parts, labeled (a)–(h). However, you get to omit two parts. Only 6 of the parts will be graded. Each part is worth 10 points. You MUST say which two parts you want to omit by filling in two bubbles.

OMIT: (a) ○ (b) ○ (c) ○ (d) ○ (e) ○ (f) ○ (g) ○ (h) ○

(a) Consider the sphere

$$x^2 + (y-3)^2 + (z+4)^2 = 14.$$

i. Where is the center of the sphere?

○ $(0,-3,4)$ ○ $(0,3,-4)$ ○ $(0,0,0)$ ○ $(0,3,4)$ ○ $(0,-3,-4)$

ii. The point $P = (1,5,-1)$ is on the sphere. What point on the sphere is farthest from P?

○ $(0,3,-4)$ ○ $(1,8,-5)$ ○ $(-1,-8,5)$ ○ $(2,4,1)$ ○ $(-1,1,-7)$

(b) Which curves arise as (x-, y-, or z-) traces for the surface

$$z = x^2 - y^2\,?$$

Indicate **all** correct answers.

○ circles ○ ellipses ○ parabolas ○ hyperbolas ○ intersecting lines

(c) Simplify each expression.

i. $(\mathbf{i} \times \mathbf{i}) \times \mathbf{j}$

○ $\mathbf{0}$ ○ $\mathbf{k}$ ○ $-\mathbf{k}$ ○ $\mathbf{j}$ ○ $-\mathbf{j}$

ii. $\mathbf{i} \times (\mathbf{i} \times \mathbf{j})$

○ $\mathbf{0}$ ○ $\mathbf{k}$ ○ $-\mathbf{k}$ ○ $\mathbf{j}$ ○ $-\mathbf{j}$

(d) Let $P = (3, -4, 12)$.

i. How far is P from the origin?

○ 5 ○ 11 ○ 13 ○ 15 ○ 19

ii. How far is P from the xy-plane?

○ 5 ○ 7 ○ 11 ○ 12 ○ $\sqrt{19}$

iii. How far is P from the z-axis?

○ 5 ○ 7 ○ 11 ○ 12 ○ $\sqrt{19}$

(e) At what point does the line

$$\frac{x}{3} = \frac{y+2}{3} = \frac{z+2}{1}$$

intersect the xy-plane?

○ $(0, -2, 0)$ ○ $(0, 2, 0)$ ○ $(3, 1, 0)$ ○ $(3, 3, 0)$ ○ $(6, 4, 0)$

(f) In how many points does the line with parametric equations

$$x = 3t + 2, \qquad y = 4t + 1, \qquad z = 5t + 2$$

intersect the hyperboloid of 1 sheet

$$x^2 + y^2 - z^2 = 1\,?$$

○ 0 ○ 1 ○ 2 ○ 4 ○ infinitely many

(g) Let

$$\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{b} = 3\mathbf{i} + \mathbf{k}.$$

Find the vector projection of $\mathbf{a}$ onto $\mathbf{b}$.

Put your answer on the blank line.
Put your work in the space below.

Answer: _____

(h) The force $\mathbf{F}$ with magnitude 3 Newtons is applied at point P and generates a torque about the origin O. (See the diagram.) The magnitude of the torque $\vec{\tau}$ has the form

$$|\vec{\tau}| = A\sin(a).$$

Find A and a.

○	$A = 12$	○	$a = 30°$
○	$A = 12\sqrt{2}$	○	$a = 45°$
○	$A = 48$	○	$a = 90°$
○	$A = 7\sqrt{2}$	○	$a = 105°$
○	$A = 4\sqrt{2}$	○	$a = 135°$