SM 223 Test #2 19 Oct 2009 Last Name ________________ Period ○ **1st** ○ **2nd** ○ **3rd**

Instructions

0. Failure to follow instruction can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are **9** problems altogether. Relative weights are given in the table.
5. **Calculators are not allowed for Problems 8 and 9 on the last page. Tear that page off and work those problems first. When you hand in that page, you may use your calculator for the rest of the test.**
6. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
7. If a problem requests you to show your work, use the space provided to receive credit.
8. Unless otherwise indicated, leave answers in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature ____________________________

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	30	
2	30	
3	30	
4	30	
5	30	
6	35	
7	20	
8	25	
9	20	
Total	**250**	

Test Score	%	Grade
200	80	*A*
175	70	*B*
150	60	*C*
< 150	< 60	*F*

SM 223 Test #2 [Curves, Partials] 19 Oct 2009

1. (a) Complete the "limit" definition of the partial derivative of $f(x, y)$ with respect to y at the point (a, b):

$$\frac{\partial f}{\partial y}(a, b) = f_y(a, b) = \underline{\hspace{8cm}}$$

(b) Use the table of values for $T(x, y)$ to estimate $T_y(0, 0)$.

- O 2
- O 5/2
- O 2/3
- O 3
- O 5

$T(x, y)$	$y = 0$	$y = 2$	$y = 4$
$x = 0$	5	10	16
$x = 3$	7	11	15
$x = 6$	11	12	15

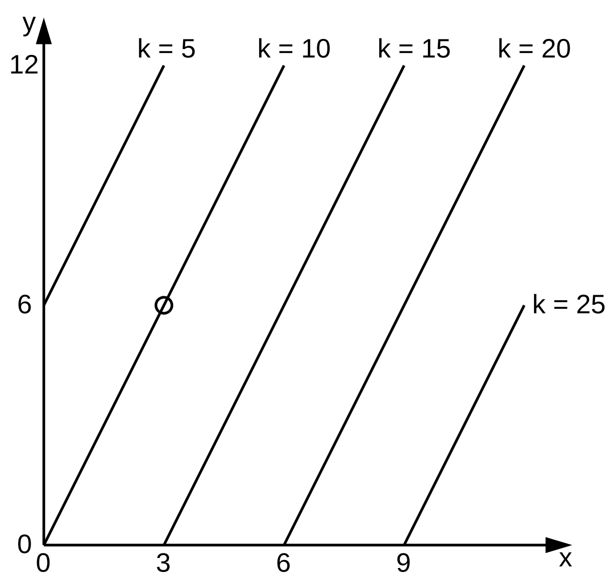
(c) The point $(3, 6)$ is shown as a small circle in the contour diagram for $f(x, y)$. Each level curve of f is a straight line of the form $f(x, y) = k$.

i. Evaluate $f(3, 6)$.

O 2　　O 3　　O 3/2　　O 5　　O 10

ii. Estimate $f_y(3, 6)$.

O $-5/6$　　O 3/2　　O 2/3　　O -5　　O 5/3



2. (a) Complete the "limit" definition of the derivative of the vector-valued function $\mathbf{r}(t)$:

$$\mathbf{r}'(t) \quad = \quad \rule{8cm}{0.4pt}$$

(b) Complete the formula for the length of the space curve $\mathbf{r}(t) = \langle x(t),\, y(t),\, z(t)\rangle$ for $t = a$ to $t = b$:

$$L = \rule{10cm}{0.4pt}$$

(c) The position of a particle at time t is given by

$$\mathbf{r}(t) \quad = \quad \langle 2t,\, \sin(t),\, \cos(t)\rangle \qquad\qquad (0 \le t \le 2\pi)$$

Find the distance traveled by the particle.

○ 4π ○ $\sqrt{14}\,\pi$ ○ $3\sqrt{2}\,\pi$ ○ $2\sqrt{5}\,\pi$ ○ $\sqrt{22}\,\pi$

3. This problem has 5 parts, labeled (a)–(e), each worth 6 points. However, you will omit two parts; only three parts will be graded. **You receive full credit for the omitted parts—but only if you fill in two bubbles in the table.**

OMIT: (a) ○ (b) ○ (c) ○ (d) ○ (e) ○

(a) The position of a particle is

$$\mathbf{r}(t) = \langle e^{-3t}, 4t \rangle.$$

Find the speed of the particle at $t = 0$.

○ 1 ○ 3 ○ 4 ○ 5 ○ 7

(b) Suppose that $\mathbf{r}'(t) = \langle 2t,\ 3t^2 \rangle$ and $\mathbf{r}(1) = \langle 1, 4 \rangle$. Find $\mathbf{r}(2)$.

○ $\langle 0, 3 \rangle$ ○ $\langle 2, 8 \rangle$ ○ $\langle 2, 10 \rangle$ ○ $\langle 4, 8 \rangle$ ○ $\langle 4, 11 \rangle$

(c) Which are level curves for the function

$$f(x, y) = x^2 - y^2\,?$$

Indicate **all** correct answers.

○ circles ○ ellipses ○ parabolas ○ hyperbolas ○ intersecting lines

(d) Which are level surfaces for the function $f(x, y, z) = x^2 - y^2 + z^2$?

- O sphere
- O ellipsoid (but not a sphere)
- O cone
- O plane
- O hyperboloid of 1 sheet
- O hyperboloid of 2 sheets

(e) The function $g(x, y)$ satisfies

$$g_x = 12x^2y^5.$$

Which function could equal g_y? (Only one answer is correct.)

- O $g_y = 20x^2y^5$
- O $g_y = 4x^3y^5$
- O $g_y = 6x^2y^3$
- O $g_y = 20x^3y^4$
- O $g_y = 12x^3y^2$

4. Fill in a bubble in each row.

TRUE	FALSE	
O	O	The graph of $f(x, y, z)$ is a surface in three dimensions.
O	O	The contour diagram of $f(x, y, z)$ is a family of surfaces in three dimensions.
O	O	Most familiar functions of two variables satisfy $f_{xy} = f_{yx}$.
O	O	If a particle's speed is constant, then its acceleration vector is $\mathbf{0}$.
O	O	If a particle's velocity is constant, then its acceleration vector is $\mathbf{0}$.
O	O	For uniform circular motion, the velocity vector is perpendicular to the position vector.

5. A particle's position at time t is given by

$$\mathbf{r}(t) = (3\cos(2t))\,\mathbf{i} \;+\; (3\sin(2t))\,\mathbf{j} \;+\; (4t)\,\mathbf{k}.$$

(a) At what time is the particle at the point $P = (0, 3, \pi)$?

○ $t = 0$ ○ $t = \pi/8$ ○ $t = \pi/4$ ○ $t = \pi/2$ ○ $t = \pi$

(b) The tangent line to the curve at point $P = (0, 3, \pi)$ has equations

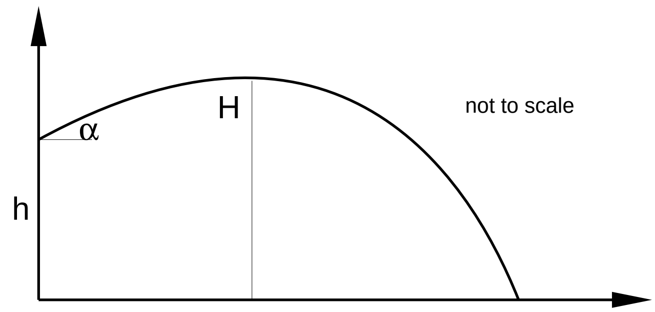
$$x = x(t), \quad y = y(t), \quad z = z(t).$$

Fill in one bubble in each column for x, y, and z. Leave the other bubbles empty.

$x(t)$	$y(t)$	$z(t)$	function
○	○	○	$3 + 6t$
○	○	○	$3 - 6t$
○	○	○	$4 - 3t$
○	○	○	$4 + 3t$
○	○	○	$6t$
○	○	○	$-6t$
○	○	○	$\pi + t$
○	○	○	$\pi - t$
○	○	○	$\pi + 4t$
○	○	○	$\pi - 4t$
○	○	○	3
○	○	○	π
○	○	○	4

6. A projectile is fired from a cliff of height h with initial speed v_0 at an angle α above horizontal. The x- and y-axes are in their usual orientations with the origin at the base of the cliff. Distance is measured in meters, and time in seconds. The projectile reaches its maximum height H. This experiment occurs on a planet where the acceleration due to gravity is $\mathbf{g} = \langle 0, -g \rangle$. The equations of motion for the projectile are:

$$\begin{aligned} x(t) &= 40t \\ y(t) &= -5t^2 + 30t + 80 \end{aligned}$$

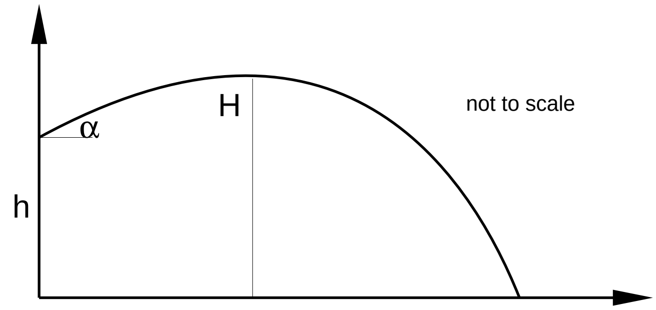


(a) Fill in the correct bubble for each parameter. Give the angle α to the nearest degree.

h	g	v_0	α	H
O 30 m	O 5 m/s^2	O 40 m/s	O 27°	O 80 m
O 40 m	O 10 m/s^2	O 50 m/s	O 37°	O 100 m
O 60 m	O 20 m/s^2	O 60 m/s	O 47°	O 120 m
O 70 m	O 30 m/s^2	O 70 m/s	O 57°	O 125 m
O 80 m	O 40 m/s^2	O 80 m/s	O 67°	O 130 m

(b) Now suppose a radio tower with height 50 meters is located 280 meters down-range. Does the projectile fall short of the tower, hit the tower, or sail over the tower?

$$x = x(t) = 40t$$
$$y = y(t) = -5t^2 + 30t + 80$$

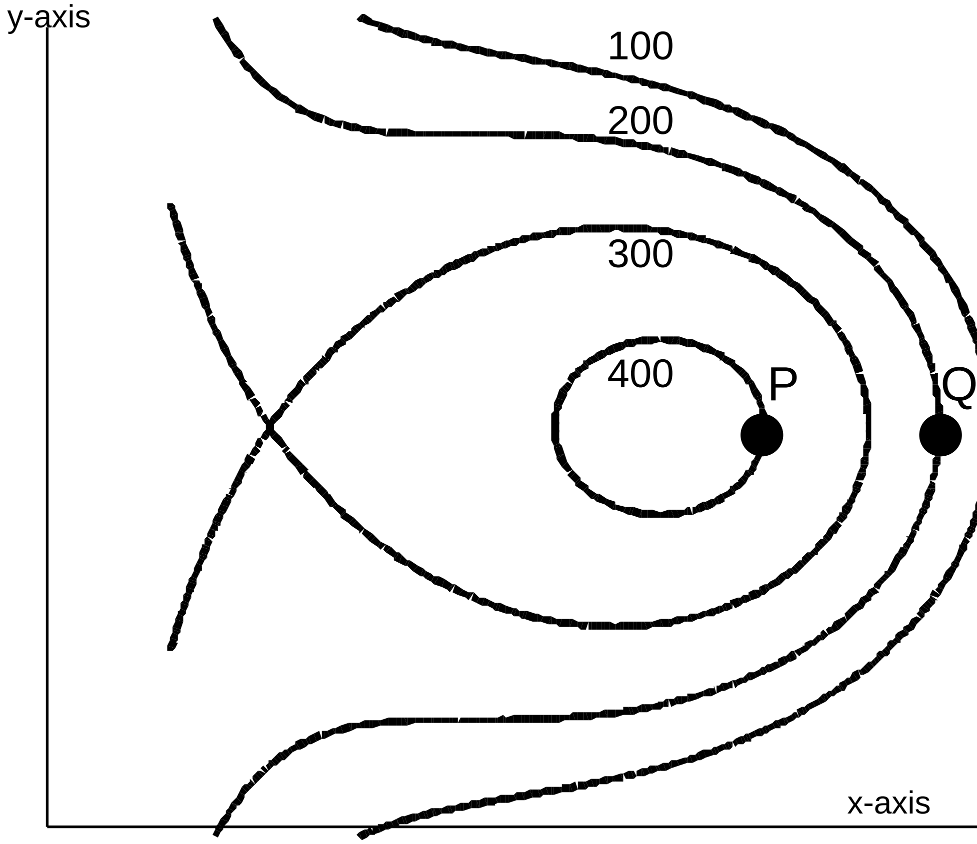


Fill in a bubble. Also indicate the value of D

- ○ Falls short: The projectile lands D meters in front of the base of tower.
- ○ Hits: The projectile hits the tower D meters from the top.
- ○ Sails over: The projectile lands D meters beyond the base of tower.

○ $D = 5$ ○ $D = 10$ ○ $D = 20$ ○ $D = 40$ ○ $D = 45$

7. The contour diagram for the function $f(x, y)$ is shown, along with two points P and Q.



Fill in a bubble in each column.

$f(P) - f(Q)$	$f_x(Q)$	$f_y(Q)$	$f_{xx}(Q)$
O positive	O positive	O positive	O positive
O negative	O negative	O negative	O negative
O zero	O zero	O zero	O zero
O undefined	O undefined	O undefined	O undefined

- $f(P) - f(Q) = 400 - 200 > 0$.
- The function decreases as we move from left to right through Q.
- Along a vertical line through Q, the function is at a maximum right at Q.
- The function is concave down in the "east-west" direction through Q since the contours are closer together to the right of Q.

8. Throughout this problem we let

$$f(x,y) = e^{x^2 y}.$$

Fill in the correct bubble. Also, show your work!

(a) Compute $f_x(5,2)$.

○ $20e^{50}$ ○ $25e^{50}$ ○ $50e^{50}$ ○ $100e^{50}$ ○ none of above; correct is ________

(b) Compute $f_y(5,2)$.

○ $20e^{50}$ ○ $25e^{50}$ ○ $50e^{50}$ ○ $100e^{50}$ ○ none of above; correct is ________

(c) Compute $f_{xy}(5,2)$.

○ $800e^{50}$ ○ $820e^{50}$ ○ $1000e^{50}$ ○ $510e^{50}$ ○ $2012e^{50}$

9. When x thousand dollars is spent on labor, and y thousand dollars is spent on advertising, a bottled water company sells

$$B(x, y) = 10x^{1/2}y^{3/2}$$

thousand gallons of water per month.

(a) How many gallons of water does the company sell when it spends $25,000 on labor and $4,000 on advertising?
Hint: $4^{3/2} = \left(4^{1/2}\right)^3 = 2^3 = 8$

○ 400,000 ○ 200,000 ○ 500,000 ○ 800,000 ○ 4,000,000

(b) Compute $B_y(25, 4)$.

○ 150 ○ 8 ○ 30 ○ 50 ○ 4

(c) **Fact:** $B_x(25, 4) = 8$.
Interpret this fact in a complete English sentence or two.

____

____

____

____

(d) Suppose the company is currently spending $25,000 on labor and $4,000 per month on advertising. If the company has budgeted an additional $1,000 to spend next month on either labor or advertising, what should it do to improve sales the most?

○ spend $1,000 on labor
○ spend $1,000 on advertising