SM 223 Test#3 9 Nov 2009 Last Name _____ **Period** ○ 1st ○ 2nd ○ 3rd

Instructions

0. Failure to follow instruction can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are **8** problems altogether. Relative weights are given in the table.
5. **Calculators are not allowed for Problems 7 and 8 on the last page. Tear that page off and work those problems first. When you hand in that page, you may use your calculator for the rest of the test.**
6. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
7. If a problem requests you to show your work, use the space provided to receive credit.
8. Unless otherwise indicated, leave answers in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature: _____

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
5	30	
6	80	
7	20	
8	20	
Total	**250**	

Test Score	%	Grade
200	80	A
175	70	B
150	60	C
< 150	< 60	F

SM 223 Test #3 [Partial Derivatives] 9 Nov 2009

1. A metal plate lies in the xy-plane with the x-axis pointing east and the y-axis pointing north. Distance is measured in meters. The temperature (Celsius) at the point (x, y) is $T(x, y)$. There is a bug at the origin $(0, 0)$.

   It is known that

   $$T(0,0) = 50, \qquad T_x(0,0) = 3, \qquad \text{and} \qquad T_y(0,0) = 4.$$

   Fill in the blanks and bubbles.

   (a) $\nabla T(0,0) =$ $\langle \quad , \quad \rangle$

   (b) Give a unit vector that points exactly northeast: $\mathbf{u} =$ $\langle \quad , \quad \rangle$

   (c) With the unit vector from above, one can show that

   $$D_{\mathbf{u}}\left(T(0,0)\right) \approx 4.9$$

   What does this number mean for the bug? Use a plain English sentence or two, understandable by a seventh-grader.

   (d) In which direction should the bug head to increase its temperature as rapidly as possible?

   - ○ parallel to the positive x-axis
   - ○ parallel to the positive y-axis
   - ○ parallel to $\langle 3/5,\ 4/5 \rangle$
   - ○ perpendicular to $\langle 3/5,\ 4/5 \rangle$
   - ○ parallel to $\langle 20,\ 12 \rangle$

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2. Complete the formulas.

   (a) The gradient of $g(x, y, z)$ is defined as

   $$\text{grad}(g) \quad = \quad \nabla g \quad = \quad \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

   (b) The directional derivative of f at point P in the direction of the unit vector $\mathbf{u}$ is

   $$D_{\mathbf{u}}\,(f(P)) \quad = \quad \underline{\qquad\qquad\qquad}$$

   (c) The equation of the tangent plane to the graph of $f(x, y)$ at the point (x_0, y_0, z_0) is

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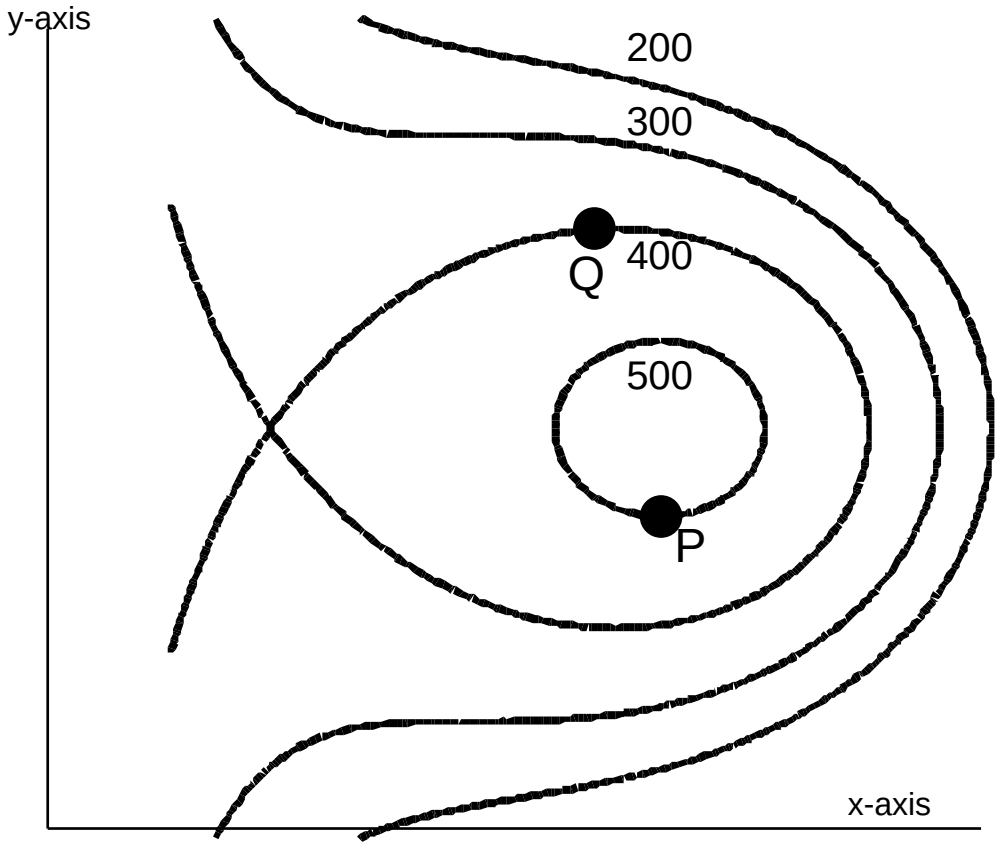
3. The radius of a cylinder is increasing at 2 m/min, and the height is decreasing at a rate of 4 m/min. Find the rate of change of the volume of the cylinder when the radius is 3 m and the height is 5 m.

   Put your answer (with units) in the blank and show your work below.

   Answer: _____

4. The contour diagram for the function $f(x, y)$ is shown, along with two points P and Q. The vector $\mathbf{u}$ is a unit vector in the direction from P to Q. Fill in a bubble in each row to indicate whether the expression is positive, negative, or zero.



+	−	0	expression
○	○	○	$f(P) - f(Q)$
○	○	○	$f_x(P)$
○	○	○	$f_y(P)$
○	○	○	$f_{xx}(P)$
○	○	○	$D_{\mathbf{u}} f(P)$

5. We are constructing a rectangular box with three faces in the coordinate planes. One vertex is at the origin, and the opposite vertex (x, y, z) is on the plane

$$5x + 2y + z = 60.$$

(a) Express the volume V of the box in terms of x and y.

$V(x, y) \quad = \quad$ _____

(b) One may show that: $V_x = y(60 - 10x - 2y)$.

Find a similar factored expression for V_y: $V_y =$ _____

(c) How many critical points does the function $V(x, y)$ have?
(The domain of V is all possible pairs (x, y), not just the positive pairs.)

○ 1 ○ 2 ○ 3 ○ 4 ○ more than 4

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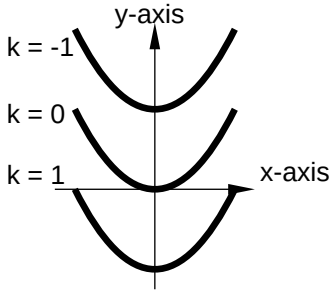
6. This problem has 8 parts, labeled (a)–(h), each worth 10 points. However, you can omit as many parts as you want. You get 5 points for each omitted part. **You must fill in the bubble in the table for any part(s) you want to omit.**

OMIT: ○ (a) ○ (b) ○ (c) ○ (d) ○ (e) ○ (f) ○ (g) ○ (h)

(a) The contour map shows the level curves $f(x, y) = k$ for $k = -1,\ 0,\ 1$. Which vector could be the gradient of f at the origin?

○ $\mathbf{i}$ ○ $-\mathbf{i}$ ○ $\mathbf{j}$ ○ $-\mathbf{j}$ ○ $\mathbf{i} + 2\,\mathbf{j}$



(b) Find the tangent plane to the graph of $f(x, y) = x^2 y$ at the point $(2, 3)$.

○ $z - 12 = 4(x - 2) + 4(y - 3)$
○ $z - 12 = 6(x - 2) + 12(y - 3)$
○ $z - 12 = 1(x - 2) + 2(y - 3)$
○ $z - 12 = 18(x - 2) + 6(y - 3)$
○ $z - 12 = 12(x - 2) + 4(y - 3)$

(c) The function $P(X, T)$ gives the pain (in units of *lejeunes*) experienced by a typical midshipman who swims X meters in a pool with temperature T degrees Fahrenheit. Identify each inequality as true or false.

TRUE	FALSE	
○	○	$P(50, 75) > P(75, 50)$
○	○	$P_X(400, 75) > 0$
○	○	$P_T(400, 75) > 0$

(d) How many critical points does the function $g(x, y) = x^2 + y^2 - 6y + 10$ have?

○ 0 ○ 1 ○ 2 ○ 3 ○ more than 3

(e) Suppose that

$$A = xy, \qquad \frac{dx}{dt} = -3 \qquad \frac{dy}{dt} = 4.$$

Find $\frac{dA}{dt}$ when $x = 5$ and $y = 6$.

○ 2 ○ 9 ○ 18 ○ 38 ○ 39

(f) Find the tangent plane to the surface $x^2 + y^2 + xe^z = 6$ at the point $(1, 2, 0)$.

○ $3x + 2y + z = 5$
○ $2x + 4y + z = 10$
○ $2x + 2y + 3z = 6$
○ $2x - 2y + 2z = 0$
○ $3x + 4y + z = 11$

(g) Let $f(x, y)$ be a function of two variables with

$$f_x(20, 10) = f_y(20, 10) = 0,$$

$$f_{xx}(20, 10) = -2 \qquad f_{yy}(20, 10) = -5, \qquad f_{xy}(20, 10) = 3.$$

How should we classify the point $(20, 10)$?

- ○ not a critical point
- ○ a saddle point
- ○ a relative minimum
- ○ a relative maximum
- ○ none of the above

(h) Estimate $f(1.1, 1.9)$ using the linear approximation (linearization) of f near $(1, 2)$, given that

$$f(1, 2) = 5, \qquad f_x(1, 2) = 3, \qquad f_y(1, 2) = 4.$$

○ 4.7 ○ 4.9 ○ 5.1 ○ 5.3 ○ 5.5

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7. For this problem we consider the point $P = (3, 1)$ and the function

$$f(x, y) = x^2 y - y.$$

(a) Find the gradient of f at P.

○ $\langle 5, 6 \rangle$ ○ $\langle 6, 8 \rangle$ ○ $\langle 5, 8 \rangle$ ○ $\langle 5, 9 \rangle$ ○ $\langle 6, 9 \rangle$

(b) Find the maximum rate of change of f at P.

○ $\sqrt{61}$ ○ 10 ○ 14 ○ $\sqrt{89}$ ○ $\sqrt{97}$

(c) Find the directional derivative of f at P in the direction from $P = (3, 1)$ to $Q = (7, 4)$.

○ $\sqrt{74}$ ○ $\sqrt{74}/5$ ○ $74/5$ ○ $48/5$ ○ 48

8. In this problem we consider the function

$$f(x,y) = 3xy - x^2y - xy^2$$

with first partial derivatives

$$f_x = 3y - 2xy - y^2 = y(3 - 2x - y),$$

$$f_y = 3x - x^2 - 2xy = x(3 - x - 2y),$$

second partial derivative

$$f_{xx} = -2y,$$

and discriminant

$$D = 4xy - (3 - 2x - 2y)^2.$$

(a) Determine whether each given point (x, y) a critical point.
Fill in a bubble YES or NO in the first two columns.

(b) **If** the point is a critical point, then classify it as relative maximum, relative minimum, or saddle point by filling in a bubble among the last three columns.

yes	no	point	max	min	saddle
○	○	$(3,1)$	○	○	○
○	○	$(0,0)$	○	○	○
○	○	$(1,1)$	○	○	○