SM 223 Test#4 9 Dec 2009 Last Name _____ **Period** ○ 1st ○ 2nd ○ 3rd

Instructions

0. Failure to follow instruction can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are **8** problems altogether. Relative weights are given in the table.
5. **Calculators are not allowed for Problems 7 and 8 on the last page. Tear that page off and work those problems first. When you hand in that page, you may use your calculator for the rest of the test.**
6. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
7. If a problem requests you to show your work, use the space provided to receive credit.
8. Unless otherwise indicated, leave answers in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature: _____

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	25	
2	25	
3	50	
4	30	
5	30	
6	35	
7	20	
8	35	
Total	**250**	

Test Score	%	**Grade**
200	80	A
175	70	B
150	60	C
< 150	< 60	F

Test 1	**250**	
Test 2	**250**	
Test 3	**250**	
Test 4	**250**	
Quizzes*	**250**	
Subtotal	**1250**	
− Lowest Test	**−250**	
16-Week Total	**1000**	

16-Week Grade	%	**Grade**
800	80	A
700	70	B
600	60	C
< 600	< 60	F

* 15 best quizzes (out of 24) times 250/150

SM 223 Test #4 [Lagrange/Multiple Integrals] **9 Dec 2009**

1. Throughout this problem we consider the function $f(x, y)$ of two variables defined for all (x, y) in a rectangle

$$R = \{(x, y) \big| \ a \le x \le b, \ c \le y \le d\}.$$

We partition the interval $[a, b]$ into m equal sub-intervals, and the interval $[c, d]$ into n equal sub-intervals.

(a) Complete our 'limit' definition of the double integral of f over R.

$$\int\int_R f(x, y)\, dA \quad = \quad \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

(b) In your definition what is the point (x^*_{ij}, y^*_{ij}) called? $\underline{\qquad\qquad\qquad\qquad}$

(c) In your definition what is ΔA?

○ $\frac{mn}{abcd}$ ○ $\frac{abcd}{mn}$ ○ $\frac{(b-a)(d-c)}{mn}$ ○ $\frac{mn}{(b-a)(d-c)}$ ○ $\frac{mn}{(d-a)(c-b)}$

(d) Complete the formula for the average value of f over R:

Average value of f over R $\quad = \quad \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$

(e) The definite integral $\int_a^b g(x)\, dx$ has a geometric interpretation: It is the (signed) area under the curve $y = g(x)$ over the interval $[a, b]$. State the corresponding geometric interpretation for the double integral

$$\int\int_R f(x, y)\, dA.$$

$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

2. We are constructing a rectangular box with three faces in the coordinate planes. One vertex is at the origin, and the opposite vertex (x, y, z) is on the plane

$$5x + 2y + z = 60.$$

We rejoice at the opportunity to use Lagrange multipliers to find the dimensions of the box with maximum volume.

(a) The volume of the box in terms of x, y, and z is $V(x, y, z) =$ _____

(b) What is our constraint? _____

(c) List the four equations in four unknowns that arise from the method of Lagrange multipliers.

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SM 223 Test #4 [Lagrange/Multiple Integrals] **9 Dec 2009**

3. **Multiple Choice.**

(a) A lamina has center of mass $(\overline{x}, \overline{y}) = (3, 5)$ and moment $M_x = 30$ about the x-axis. Find the mass.

○ $m = 6$ ○ $m = 10$ ○ $m = 15$ ○ $m = 90$ ○ $m = 150$

(b) What solid E has volume given by the triple integral

$$\iiint_E 1\, dV \quad = \quad \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{12}} 1\, dz\, dy\, dx \ ?$$

○ circle ○ sphere ○ hemisphere ○ cylinder ○ cube

(c) Use a Midpoint approximation with $m = n = 2$ to estimate

$$\int_0^4 \int_0^4 f(x, y)\, dy\, dx$$

for $f(x, y) = x^2 y$.

○ 40 ○ 80 ○ 140 ○ 160 ○ 200

(d) Evaluate the double integral,

$$\int\int_R 12(x^2+y^2)\,dA,$$

where R be the ring-shaped region between the two circles with polar equations $r = 1$ and $r = 2$.

○ 3π ○ 18π ○ 36π ○ 56π ○ 90π

(e) What shape is the region of integration for the double integral

$$\int_0^1 \int_{-x}^{x} x^2\,dy\,dx \ ?$$

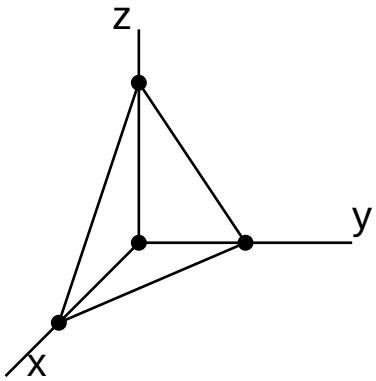
○ parabola ○ paraboloid ○ right triangle ○ non-right triangle ○ trapezoid

SM 223 Test #4 [Lagrange/Multiple Integrals] 9 Dec 2009

4. Fill in the blanks for the multiple integral expressions for the volume of the tetrahedron bounded by the three coordinate planes and the plane

$$3x + 2y + z = 12.$$

YOU ARE NOT BEING ASKED TO EVALUATE THE INTEGRALS



It is helpful to compute the intercepts $(4, 0, 0)$, $(0, 6, 0)$, and $(0, 0, 12)$.

$$\text{volume} = \int_{\_\_\_\_}^{\_\_\_\_} \int_{\_\_\_\_}^{\_\_\_\_} \int_{\_\_\_\_}^{\_\_\_\_} 1 \; dz\, dy\, dx$$

$$\text{volume} = \int_{\_\_\_\_}^{\_\_\_\_} \int_{\_\_\_\_}^{\_\_\_\_} \int_{\_\_\_\_}^{\_\_\_\_} 1 \; dx\, dy\, dz$$

SM 223 Test #4 [Lagrange/Multiple Integrals] **9 Dec 2009**

5. A quarter-circular lamina in the first quadrant is bounded by the x- and y-axes and the circle

$$x^2 + y^2 = 4.$$

The density at the point (x, y) is

$$\rho(x, y) \quad = \quad \text{distance from } (x, y) \text{ to the origin}$$

in grams per square unit.

Matching. Write a capital letter in each blank to indicate the correct double integral. Not every capital letter is used.

____ mass m of lamina

____ area A of lamina

____ moment M_x of lamina about the x-axis

____ moment M_y of lamina about the y-axis

____ moment of inertia I_x of lamina about the x-axis

____ moment of inertia I_O of lamina about the origin

A: $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

B: $\displaystyle\int_0^{\pi/2} \int_0^2 r\cos(\theta)\cdot r\cdot r\, dr\, d\theta$

C: $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} x^2\sqrt{x^2+y^2}\, dy\, dx$

D: $\displaystyle\int_0^{\pi/2} \int_0^2 r^4\, dr\, d\theta$

E: $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$

F: $\displaystyle\int_0^{\pi/2} \int_0^2 r\, dr\, d\theta$

G: $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} y^2\sqrt{x^2+y^2}\, dy\, dx$

H: $\displaystyle\int_0^{\pi/2} \int_0^2 r^3\, dr\, d\theta$

6. A student's joy is the product of the amount of money M (in dollars) he has and the amount C (in liters) of cola he has. So

$$\text{Joy} \quad = \quad J(M,C) \quad = \quad MC.$$

Currently he has \$10 and no cola. Each liter of cola costs \$0.40. The purpose of this problem is to determine how much cola the student should buy to maximize his joy. We will use Lagrange multipliers.

(a) What is the constraint?

○ $M+0.4C=10$ ○ $0.4M+C=10$ ○ $M+\frac{C}{0.4}=10$ ○ $\frac{M}{0.4}+C=10$

(b) List the three equations in three unknowns that arise from Lagrange multipliers.

_____ _____ _____

(c) Solve the system to determine how much cola the student should buy. Put your final answer in a box.

SM 223 Test#4 9 Dec 2009 Last Name _____ **Period** ○ 1st ○ 2nd ○ 3rd

7. (a) Fill in the correct bubble and show your work for full or partial credit. Evaluate the iterated integral

$$\int_0^3 \int_0^4 x^2 y \, dy \, dx.$$

○ 12 ○ 18 ○ 24 ○ 36 ○ 72

(b) A sprinkler distributes water in a circular pattern. It supplies water to a depth of $f(r)$ feet per hour at each point at distance r from the sprinkler.

i. Interpret the value of

$$\frac{1}{\pi \cdot 12^2} \int_0^{2\pi} \int_0^{12} f(r) \, dr \, d\theta$$

in words understandable by a middle school student.

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_____

ii. Which one of the following functions is reasonable for $f(r)$?

○ r^2 ○ $12r$ ○ $e^{r/12}$ ○ e^{12r} ○ $e^{-r/12}$

8. Throughout this problem we consider the iterated integral

$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx.$$

(a) The region of integration is roughly triangular. One of the three corners is $(0,0)$. Fill in *two* bubbles for the other two corners.

○ $(2,0)$ ○ $(4,0)$ ○ $(0,2)$ ○ $(0,4)$ ○ $(2,4)$ ○ $(4,2)$

(b) Rewrite the iterated integral in the reverse order of integration:

$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx \quad = \quad \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \underline{\qquad\qquad\qquad\qquad} \, dx\, dy$$

(c) Evaluate the iterated integral

$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx.$$

Fill in the correct bubble and show your work for full or partial credit.

○ e^8 ○ $e^8 - 1$ ○ $\frac{e^8-1}{3}$ ○ $3e^8$ ○ none of above