SM 223_{TSM} Test #2 [18 Oct 2010] **Last Name**________________________ **Section:** ○ **4041** ○ **5041** ○ **6041**

Instructions

0. Failure to follow instruction can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are $\boxed{8}$ problems altogether. Relative weights are given in the table.
5. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
6. If a problem requests you to show your work, use the space provided to receive credit.
7. **Calculators are forbidden for the last page of the test. Do that page first. You must hand in that page before putting your calculator on your desk.**
8. Unless otherwise indicated, leave answer in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature ____________________________

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	50	
2	35	
3	45	
4	30	
5	35	
6	45	
7	30	
8	30	
Total	**300**	

Test Score	%	Grade
240	80	A
210	70	B
180	60	C
< 180	< 60	F

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1. The velocity of a particle at time t is

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 3t^2,\ 4t\rangle.$$

(a) Find the speed at time $t = 1$.

○ 5 ○ 7 ○ $\sqrt{2}$ ○ $2\sqrt{10}$ ○ $\sqrt{5}$

(b) Which definite integral represents the total distance traveled (length of the curve) of the particle between $t = 1$ and $t = 2$?

○ $\int_1^2 \sqrt{36t^2 + 16}\, dt$
○ $\int_1^2 \sqrt{3t^2 + 4t}\, dt$
○ $\int_1^2 t\sqrt{9t^2 + 16}\, dt$
○ $\int_1^2 \sqrt{3t^2 + 4}\, dt$
○ $\int_1^2 \sqrt{3}\, t + 2\sqrt{t}\, dt$
○ $\int_1^2 \sqrt{3t^4 + 4t^2}\, dt$

(c) Suppose we know that the particle's position at time $t = 1$ is $\mathbf{r}(1) = \langle 7,\ 0\rangle$. Find the position at $t = 2$.

○ $\langle 8, 8\rangle$ ○ $\langle 14, 6\rangle$ ○ $\langle 14, 8\rangle$ ○ $\langle 8, 6\rangle$ ○ $\langle 4, 12\rangle$

(d) Continue to assume that $\mathbf{r}(1) = \langle 7,\ 0\rangle$.
Find the the parametric equations $x(t)$ and $y(t)$ for the tangent line to the particle's trajectory at $t = 1$.

$x(t)$: ○ $7t$ ○ $7 + 3t$ ○ $3 + 7t$ ○ $6 + 3t$ ○ $6 + 7t$

$y(t)$: ○ $3t$ ○ $4 + 3t$ ○ $3 + 4t$ ○ $-2 + 4t$ ○ $4t$

2. (a) State our formula for the tangent plane to the graph $z = f(x, y)$ at the point with $(x, y) = (a, b)$.

____

(b) Which two of the listed points lie on the tangent plane to the surface

$$z = f(x, y) = 2x^2 + y^2$$

at the point with $(x, y) = (1, 1)$? FILL IN TWO BUBBLES

○ $(1, 1, 3)$ ○ $(1, 1, 4)$ ○ $(2, 2, 12)$ ○ $(0, 0, -3)$ ○ $(0, 0, 3)$ ○ $(0, 0, 0)$

(c) A function $g(x, y)$ satisfies

$$g(20, 12) = 3, \qquad g_x(20, 12) = 4, \qquad g_y(20, 12) = 5.$$

Use a linearization to estimate $g(20.1, 11.8)$.

○ 3.1 ○ 3.9 ○ 3.0 ○ 2.8 ○ 2.4

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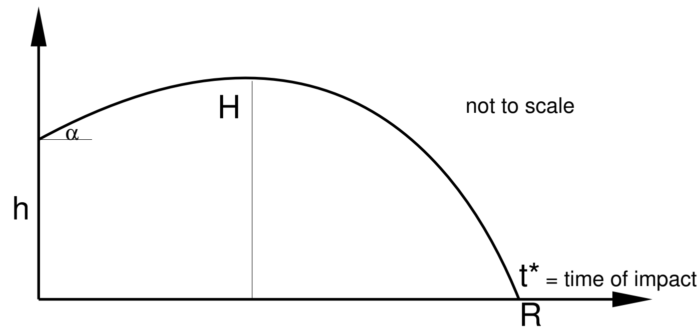
3. A projectile is fired from a cliff of height h with initial speed v_0 at an angle α above horizontal. The x- and y-axes are in their usual orientations with the origin at the base of the cliff. Distance is measured in meters, and time in seconds. The projectile reaches a maximum height H. At time $t = t^*$ the projectile hits the level land below at a distance R from the base of the cliff. The minimum speed of the projectile during its flight is v_{min}, and the speed at impact is v^*. This experiment occurs on a planet where the acceleration due to gravity is $\mathbf{g} = \langle 0, -g \rangle$. (Do not assume that $g = 9.8$.) The projectile's motion is given by:

$$\begin{aligned} x(t) &= 25t \\ y(t) &= -10t^2 + 60t + 400 \\ &= -10(t+4)(t-10). \end{aligned}$$



(a) Fill in the correct bubble for each parameter.
Give the angle α to the nearest degree.

h	g	v_0	α	H
○ 25 m	○ 5 m/s^2	○ 45 m/s	○ 27°	○ 425 m
○ 60 m	○ 10 m/s^2	○ 50 m/s	○ 37°	○ 470 m
○ 250 m	○ 20 m/s^2	○ 60 m/s	○ 47°	○ 475 m
○ 400 m	○ 25 m/s^2	○ 65 m/s	○ 57°	○ 490 m
○ 800 m	○ 60 m/s^2	○ 130 m/s	○ 67°	○ 525 m

(b) This is a continuation of the same projectile problem.
The data and diagram are repeated on this page for your convenience.

A projectile is fired from a cliff of height h with initial speed v_0 at an angle α above horizontal. The x- and y-axes are in their usual orientations with the origin at the base of the cliff. Distance is measured in meters, and time in seconds. The projectile reaches a maximum height H. At time $t = t^*$ the projectile hits the level land below at a distance R from the base of the cliff. The minimum speed of the projectile during its flight is $v_{\min}$, and the speed at impact is v^*. This experiment occurs on a planet where the acceleration due to gravity is $\mathbf{g} = \langle 0, -g \rangle$. (Do not assume that $g = 9.8$.) The projectile's motion is given by:

$$
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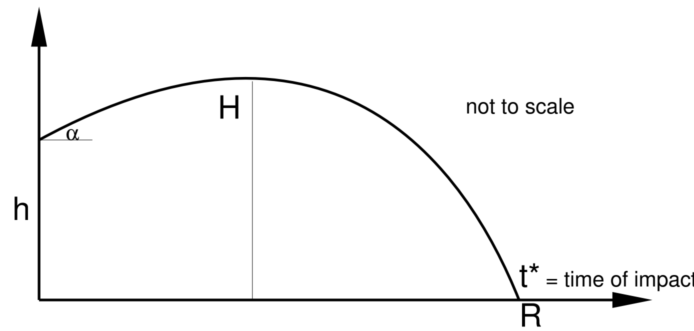
$$



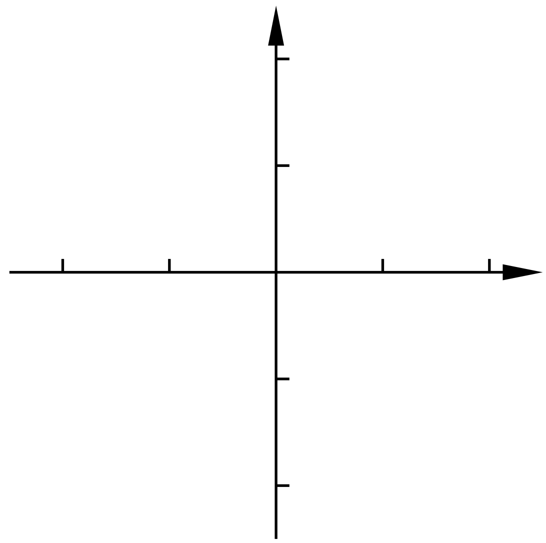
Fill in the correct bubble for each parameter.
Give the speed at impact v^* to the nearest m/sec.

$v_{\min}$	t^*	R	v^*
O 0 m/s	O 4 s	O 225 m	O 65 m/s
O 25 m/s	O 6 s	O 250 m	O 127 m/s
O 35 m/s	O 10 s	O 300 m	O 135 m/s
O 40 m/s	O 12 s	O 320 m	O 137 m/s
O 60 m/s	O 13 s	O 490 m	O 142 m/s

4. This problem deals with the function

$$f(x, y) = x^2 + y^2.$$

(a) Draw a contour diagram for $f(x, y)$.
Include <u>and label</u> the contours for $k = 0, 1, 2, 3,$ and 4.



(b) The graph of $f(x, y)$ is a

○paraboloid ○cone ○hemisphere ○sphere ○hyperbolic paraboloid

5. This problem is about the uniform circular motion defined by the vector-valued function

$$\mathbf{r}(t) = \langle R\cos(\omega t),\ R\sin(\omega t)\rangle,$$

where R and ω ("omega") are positive constants.

(a) $|\mathbf{r}(t)| =$

○ R ○ ω ○ $R\omega$ ○ R/ω ○ $R\omega^2$ ○ none of above

(b) $|\mathbf{r}'(t)| =$

○ R ○ ω ○ $R\omega$ ○ R/ω ○ $R\omega^2$ ○ none of above

(c) Complete the statement of the theorem by writing one word in the blank. Then give a proof of the theorem, as we did in class.

Theorem. The position vector $\mathbf{r}(t)$ and velocity vector $\mathbf{r}'(t)$ are always __________

Proof.

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NO CALCULATOR ALLOWED FOR THIS PAGE

FILL IN YOUR NAME AND SECTION NUMBER.
Detach this sheet from the rest of the test.
Solve the problems without using your calculator.
After you hand in this sheet of paper, you can use your calculator.

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6. Throughout this problem we let

$$f(x,y) = e^{x^2+3xy}.$$

Fill in the correct bubble. Also, show your work!

(a) Compute $f_x(5,2)$.

○ $10e^{55}$ ○ $15e^{55}$ ○ $16e^{55}$ ○ $55e^{55}$ ○ none of above; correct is __________

(b) Compute $f_y(5,2)$.

○ $15e^{55}$ ○ $16e^{55}$ ○ $50e^{55}$ ○ $100e^{55}$ ○ none of above; correct is __________

(c) Compute $f_{xy}(5,2) - f_{yx}(5,2)$.

○ $10e^{55}$ ○ $12e^{55}$ ○ $15e^{55}$ ○ $16e^{55}$ ○ none of above; correct is __________

7. The function $P(X,T)$ gives the pain (in units of *lejeunes*) experienced by a typical midshipman who swims X meters in a pool with temperature T degrees Fahrenheit.

   (a) Identify each inequality as true or false.

TRUE	FALSE	
O	O	$P(50,75) > P(75,50)$
O	O	$P_X(400,75) > 0$
O	O	$P_T(400,75) > 0$

   (b) What are the units for $P_X(400,75)$? ______________

8. (a) Complete the "limit" definition of the partial derivative of $f(x,y)$ with respect to x at the point (a,b):

$$\frac{\partial f}{\partial x}(a,b) = f_x(a,b) = \underline{\hspace{8cm}}$$

   (b) Use the table of values for $T(x,y)$ to estimate $T_x(0,0)$.

   - O 2
   - O 5/2
   - O 2/3
   - O 3
   - O 5
   - O none of above; correct is ____

$T(x,y)$	$y=0$	$y=2$	$y=4$
$x=0$	5	10	16
$x=3$	7	11	15
$x=6$	11	12	15