SM 223_{TSM} Test #1 [17 Sep 2012] Last Name________________ Section: ○ 4023 ○ 5023 ○ 6023

Instructions

0. Failure to follow instructions can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page,** except your signature on the line below to indicate you've read and understood the directions.
4. There are $\boxed{8}$ problems altogether. Point values are given in the table.
5. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
6. If a problem requests you to show your work, use the space provided to receive credit.
7. **Calculators are forbidden for the last page of the test only. Do that page first. You must hand in that page before using your calculator.**
8. Unless otherwise indicated, leave answer in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature ________________________

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	25	
2	25	
3	50	
4	50	
5	25	
6	25	
7	60	
8	40	
Total	**300**	
Test Score	%	**Grade**
240	80	*A*
210	70	*B*
180	60	*C*
< 180	< 60	*F*

SM 223 **Test #1** **[17 Sep 2012]**

1. This problem deals with two three-dimensional vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ in a tail-to-tail position forming an angle θ. Assume that neither vector is a scalar multiple of the other.

   (a) Complete our geometric definition of the cross product:
   The cross product of $\overrightarrow{a}$ and $\overrightarrow{b}$ is

   $$\overrightarrow{a} \times \overrightarrow{b} = (\underline{\hspace{4cm}})\,\overrightarrow{n},$$

   where $\overrightarrow{n}$ is the unit normal vector determined by the right-hand rule.

   (b) Draw a big sketch of the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ and the parallelogram they determine.

   (c) Write down a formula (involving the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$) for the area of the parallelogram.

   $$\text{Area of parallelogram} = \underline{\hspace{4cm}}$$

   (d) Explain why the formula is true, as we did in class. You will likely need to insert further items in your big sketch in (b).

2. This problem deals with the point $P = (1, 2, 12)$ and the plane

$$2x + 2y + z = 36.$$

(a) Give parametric equations for the line through P perpendicular to the plane.

____

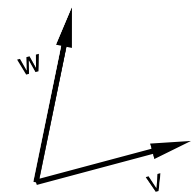
(b) Find the point on the plane that is closest to P.

Put your answer here: (, ,)

Show your work below.

3. The sketch shows two vectors $\mathbf{v}$ and $\mathbf{w}$. The vectors satisfy

$$\boxed{|\mathbf{v}| = 4 \qquad |\mathbf{w}| = 6 \qquad \mathbf{v}\cdot\mathbf{w} = 7}$$



Note: The figure is not precisely to scale.

(a) Draw the vector projection of $\mathbf{v}$ onto $\mathbf{w}$ in the sketch at a suitable position.

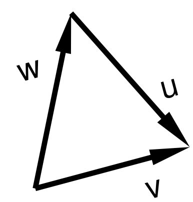
(b) Find $\mathbf{v}\cdot\mathbf{v}$.

○ 0 ○ 2 ○ 4 ○ 7 ○ 16

(c) Find $\mathbf{w}\cdot\mathbf{v}$.

○ -7 ○ 7 ○ 0 ○ -24 ○ 24

(d) Which expression is equal to the vector $\mathbf{u}$ in the diagram?



○ $\mathbf{v}+\mathbf{w}$ ○ $\mathbf{v}-\mathbf{w}$ ○ $\mathbf{w}-\mathbf{v}$ ○ $\left(\frac{1}{|\mathbf{v}|}\right)\mathbf{v}$ ○ $\left(\frac{1}{|\mathbf{w}|}\right)\mathbf{w}$

(e) Find the length of the vector $\mathbf{u}$.

○ cannot be determined from the given information
○ 6
○ $\sqrt{38}$
○ 7
○ $4\sqrt{3}$

4. Consider the three points

$$P = (1, 3, 3), \qquad Q = (4, 5, 9), \qquad R = (2, 5, 5).$$

Useful facts:

$$\boxed{\overrightarrow{PQ} = \langle 3, 2, 6 \rangle, \quad |\overrightarrow{PQ}| = 7, \quad |\overrightarrow{PR}| = 3, \quad \overrightarrow{PQ} \cdot \overrightarrow{PR} = 19, \quad \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -8, 0, 4 \rangle.}$$

(a) Give parametric equations of the line through P and Q.

____

(b) Give an equation of the plane through P, Q, and R:

____

(c) Find the area of $\triangle PQR$. Round your answer to the nearest 0.1.

O 8.9 O 4.5 O 4.4 O 2.2 O 2.1

(d) Find the angle formed by vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Round your answer to the nearest degree.

O $22°$ O $25°$ O $28°$ O $31°$ O $35°$

(e) Find the work done by the constant force $\overrightarrow{PQ}$ in moving an object on a straight line from P to R.

Note: The distance is in meters and force is in Newtons. So the work is in Joules.

O 29 J O 19 J O 80 J O $\sqrt{80}$ J O 4 J

5. Identify each expression as VECTOR, SCALAR, or NONSENSE.
   Fill in one bubble in each row.

VECTOR	SCALAR	NONSENSE	
○	○	○	$\mathbf{i} \times \mathbf{k}$
○	○	○	$\langle \mathbf{i}, \mathbf{j}, \mathbf{k} \rangle \cdot \langle \mathbf{i}, -\mathbf{j}, \mathbf{k} \rangle$
○	○	○	$\left(\overrightarrow{a} \cdot \overrightarrow{b} \right) \times \overrightarrow{c}$
○	○	○	$(\overrightarrow{v} \times \overrightarrow{w}) \times \overrightarrow{u}$
○	○	○	$\left(\overrightarrow{a} \cdot \overrightarrow{b} \right) \overrightarrow{c}$

6. TRUE or FALSE. Fill in one bubble in each row.
   All statements deal with three dimensions.

TRUE	FALSE	
○	○	Two lines parallel to a third line must be parallel to each other.
○	○	Two lines perpendicular to a third line must be parallel to each other.
○	○	Two lines perpendicular to a plane are parallel to each other.
○	○	Two lines either intersect or are parallel.
○	○	A plane and a line either intersect or are parallel.

7. This problem has 6 parts, labeled (a)–(f). Each part is worth 10 points. However, you can omit as many parts as you want. You automatically receive 5 points for each part you omit.

   You MUST fill in the "OMIT" bubble to get the 5 points.

   (a) O OMIT

   Which curves arise as (x-, y-, or z-) traces for the surface

   $$z = x^2 - y^2\,?$$

   Give **all** correct answers.

   O circles O (non-circular) ellipses O parabolas O hyperbolas O intersecting lines

   (b) O OMIT

   Identify each surface.

   i. $z = x^2 - y^2$

   O ellipsoid O paraboloid O hyperboloid of 1 sheet O hyperbolic paraboloid O cone

   ii. $\langle 20, 15, 0\rangle \cdot \langle x, -y, z\rangle = 15$

   O sphere O (non-spherical) ellipsoid O cone O plane O none of above

   (c) O OMIT

   Fill in the bubble for **every** unit vector.

   O $\langle 1, 1\rangle$

   O $\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$

   O $-\mathbf{k}$

   O $\langle \cos(15°), \sin(15°)\rangle$

(d) ○ OMIT

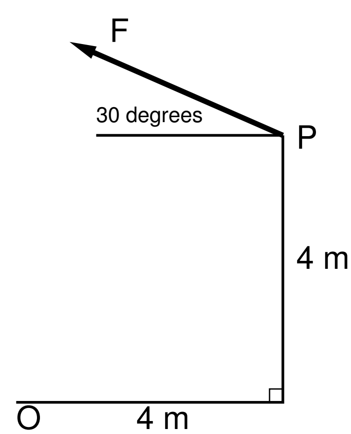
The diagram shows a force $\overrightarrow{F}$ with magnitude 3 Newtons being applied at point P, generating a torque about the origin O through the L-shaped metal bar with arms of length 4 m.

i. The magnitude of the torque $\overrightarrow{\tau}$ has the form

$$|\overrightarrow{\tau}| = A\sin(a) \qquad \text{in units of meters-Newtons.}$$

Find A and a.

○ $A = 12$	○ $a = 30°$
○ $A = 12\sqrt{2}$	○ $a = 45°$
○ $A = 48$	○ $a = 90°$
○ $A = 7\sqrt{2}$	○ $a = 105°$
○ $A = 4\sqrt{2}$	○ $a = 135°$



ii. The direction of the torque vector $\overrightarrow{\tau}$ is

- ○ in the same direction as $\overrightarrow{OP}$
- ○ in the opposite direction as $\overrightarrow{OP}$
- ○ in the same direction as $\overrightarrow{F}$
- ○ out of the page
- ○ into the page

(e) O OMIT

This problem deals with the two lines

$$\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z}{2} \qquad \text{and} \qquad \mathbf{r}(t) = \langle 2,0,2\rangle + t\langle -1,1,0\rangle.$$

The lines intersect at the point $(2, 0, 2)$.
Thus there is a plane that contains both lines. Which one of the following vectors is perpendicular to the plane?

O $\langle 1,1,0\rangle \times \langle 2,0,2\rangle$
O $\langle 1,-1,2\rangle \times \langle 2,0,2\rangle$
O $\langle 1,1,0\rangle \times \langle -1,1,0\rangle$
O $\langle 1,-1,2\rangle \times \langle -1,1,0\rangle$
O $\langle 1,1,0\rangle \ + \ \langle 2,0,2\rangle$

(f) O OMIT

Consider the sphere

$$x^2 + (y-3)^2 + (z+4)^2 = 14.$$

i. Where is the center of the sphere?

O $(0,-3,4)$ O $(0,3,-4)$ O $(0,0,0)$ O $(0,3,4)$ O $(0,-3,-4)$

ii. The point $P = (1, 5, -1)$ is on the sphere. What point on the sphere is farthest from P?

O $(0,3,-4)$ O $(1,8,-5)$ O $(-1,-8,5)$ O $(2,4,1)$ O $(-1,1,-7)$

SM 223$_{\text{TSM}}$ Test #1 [17 Sep 2012] Last Name ________________ **Section:** ○ **4023** ○ **5023** ○ **6023**

NO CALCULATOR ALLOWED FOR THIS PAGE

FILL IN YOUR NAME AND SECTION NUMBER.
Detach this sheet from the rest of the test.
Solve the problems without using your calculator.
After you hand in this sheet of paper, you can use your calculator.

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8. This problem deals with the two vectors

$$\mathbf{a} = 3\mathbf{i} + \mathbf{k} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}.$$

Compute the following expressions.

(a) $2\mathbf{a} - \mathbf{b}$ **Answer:** __________

(b) $|\mathbf{a}|$ **Answer:** __________

(c) A unit vector in the same direction as $\mathbf{a}$ **Answer:** __________

(d) $\mathbf{a} \times \mathbf{b}$ **Answer:** __________

(e) The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ **Answer:** __________