

Instructions

0. Failure to follow instructions can result in your losing points.
1. **Do NOT turn the page or begin until instructed to do so.**
2. Print your name and indicate your section above.
3. **Write nothing else on this cover page**, except your signature on the line below to indicate you've read and understood the directions.
4. There are **8** problems altogether. Relative weights are given in the table.
5. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
6. If a problem requests you to show your work, use the space provided to receive credit.
7. **Calculators are forbidden for the last page of the test. Do that page first. You must hand in that page before putting your calculator on your desk.**
8. Unless otherwise indicated, leave answer in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature _____

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	30	
2	30	
3	40	
4	30	
5	30	
6	80	
7	30	
8	30	
Total	300	

Test 1	300	
Test 2	300	
Test 3	300	
Quizzes*	400	
12-Week Total	1300	
12-Week	Grade	%
1040		80
910		70
780		60
715		55
< 715		< 60
		Grade
		<i>A</i>
		<i>B</i>
		<i>C</i>
		<i>D</i>
		<i>F</i>

* 8 best quizzes (out of 12) times 5

Test Score	%	Grade
240	80	<i>A</i>
210	70	<i>B</i>
180	60	<i>C</i>
< 180	< 60	<i>F</i>

1. Complete the formulas.

(a) The gradient of $f(x, y, z)$ is defined as

$$\text{grad}(f) = \nabla f = \underline{\hspace{15em}}$$

(b) The formula for the directional derivative of f at point P in the direction of the unit vector \mathbf{u} is

$$D_{\mathbf{u}}(f(P)) = \underline{\hspace{10em}}$$

(c) Fill in the blanks with mathematical expressions to complete the statement of the theorem.

Let $f = f(x, y, z)$ be a function of three variables and let P be a point.

i. *The maximum rate of change of f at P occurs in the direction* $\underline{\hspace{5em}}$

ii. *The maximum rate of change of f at P equals* $\underline{\hspace{5em}}$

(d) Complete the proof of both parts of the Theorem.

Proof. Let \mathbf{u} be a unit vector. The rate of change of f at P in the direction \mathbf{u} equals the directional derivative $D_{\mathbf{u}}(f(P))$. From the formula in (b), this directional derivative is

$$D_{\mathbf{u}}(f(P)) =$$

SM 223 Test #3 [8 Nov 2010]

2. This problem is about a cylinder with radius r and height h .

(a) Complete the formula for the volume of the cylinder. $V =$ _____

(b) The radius of a certain cylinder is always increasing at the rate of 2 m/hr, but the volume remains constant at $48\pi \text{ m}^3$.

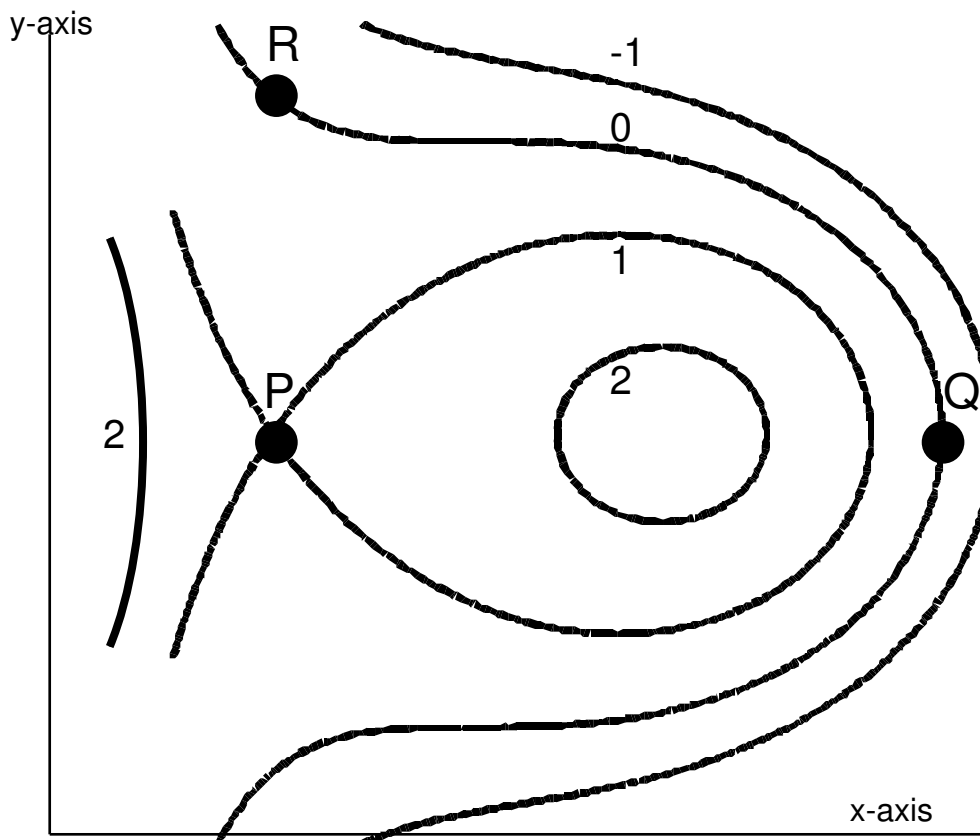
At a certain instant the radius of the cylinder is 4 m.

What can we say about how the height of the cylinder is changing at this instant?

FILL IN A BUBBLE—AND A BLANK, IF APPROPRIATE.

- We cannot say anything definitive
- The height is not changing
- The height is decreasing at some rate we cannot determine
- The height is increasing at some rate we cannot determine
- The height is decreasing at the rate _____
- The height is increasing at the rate _____

3. The contour diagram for the function $f(x, y)$ is shown with points P , Q , and R . The vector \mathbf{u} is a unit vector in the direction from Q to R . Identify each expression as positive, negative, or zero. Fill in a bubble in each row.



+	-	0	expression
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f(P)$
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f_x(P)$
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f_{xx}(P)$
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f_{xx}(P)f_{yy}(P) - [f_{xy}(P)]^2$

+	-	0	expression
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f_x(Q)$
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$f_y(Q)$
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$ \nabla f(Q) - \nabla f(R) $
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$D_{\mathbf{u}}f(Q)$

SM 223 Test #3 [8 Nov 2010]

4. In this problem we find the minimum distance between the point $(8, 6, 0)$ and the cone $z^2 = x^2 + y^2$.

(a) Express the *square of the distance* between $(8, 6, 0)$ and the point (x, y, z) on the cone as a function $f(x, y)$ of x and y .

IS IS BEST IF YOU DO NOT EXPAND OR OTHERWISE "SIMPLIFY" YOUR ANSWER.

$f(x, y) =$ _____

(b) The minimum distance between $(8, 6, 0)$ and the cone is

- 5 $5\sqrt{2}$ 10 100 $5\sqrt{3}$ none of above; correct is _____

FILL IN A BUBBLE AND SHOW YOUR WORK BELOW

SM 223 Test #3 [8 Nov 2010]

5. We are constructing a open-top aquarium with glass sides and a slate base. The volume of the aquarium must be 5000 cm^3 . Glass costs 3 cents/cm², and slate costs 7 cents/cm². The height of the aquarium is z , and the base has length x and width y .

(a) Express the total cost of the aquarium in terms of x , y , and z .

$$C(x, y, z) = \underline{\hspace{4cm}}$$

(b) What is our constraint on x , y , and z ?

$$\underline{\hspace{4cm}}$$

(c) List the four equations in four unknowns that arise when the method of Lagrange multipliers is used to minimize the cost of the aquarium.

DO NOT SOLVE THE SYSTEM.

$\underline{\hspace{4cm}}$

$\underline{\hspace{4cm}}$

$\underline{\hspace{4cm}}$

$\underline{\hspace{4cm}}$

6. This problem has 8 parts, labeled (a)–(h), each worth 10 points. However, you can omit as many parts as you want. You get 5 points for each omitted part.

You must fill the bubble in the table for any part(s) you want to omit.

OMIT: (a) (b) (c) (d) (e) (f) (g) (h)

(a) Let $f(x, y)$ satisfy

$$f_x(20, 10) = f_y(20, 10) = 0,$$

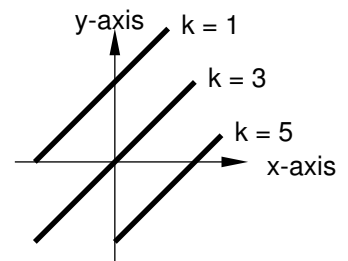
$$f_{xx}(20, 10) = 2 \quad f_{yy}(20, 10) = 5, \quad f_{xy}(20, 10) = -3.$$

How should we classify the point $(20, 10)$?

- not a critical point
- a saddle point
- a relative minimum
- a relative maximum
- none of the above

(b) The contour map shows the level curves $f(x, y) = k$ for $k = 1, 3, 5$. Which vector could be the gradient of f at the origin?

- $\mathbf{i} + \mathbf{j}$ $2\mathbf{i} + 2\mathbf{j}$ $3\mathbf{i} + 3\mathbf{j}$ $\mathbf{i} - \mathbf{j}$ $-\mathbf{i} + \mathbf{j}$ $-\mathbf{i} - \mathbf{j}$



SM 223 Test #3 [8 Nov 2010]

(c) How many critical points does the function $g(x, y) = (x + y)^2$ have?

- 0 1 2 3 more than 3, but finite infinitely many

(d) How many critical points does the function $F(x, y) = e^x \cos(y)$ have?

- 0 1 2 3 more than 3, but finite infinitely many

(e) The function $f(x, y) = -20x + 13y$ has no critical points. What is the minimum value of f for (x, y) in (or on the boundary of) the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$?

- 0 -20 -7 -13 -33 none of above

SM 223 Test #3 [8 Nov 2010]

(f) Suppose $w = f(x, y, z, t)$, and $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, and $t = t(u, v)$. What is the *total* number of nodes in the corresponding tree diagram? (The nodes are the points we draw to represent the variables.)

- 7
- 9
- 11
- 13
- 15

(g) Suppose that

$$z = x^2y + 3xy^4, \quad \text{and} \quad x = \sin(2t), \quad \text{and} \quad y = \cos(t).$$

Find $\frac{dz}{dt}$ when $t = 0$.

- 0
- 2
- 4
- 6
- 8

(h) Find the tangent plane to the surface $y = x^2 - z^2$ at the point $(4, 7, 3)$.

- $8(x - 4) - (y - 7) - 6(z - 3) = 0$
- $8(x - 4) + (y - 7) - 6(z - 3) = 0$
- $-4(x - 4) - 7(y - 7) - 3(z - 3) = 0$
- $4(x - 4) + 0(y - 7) + 3(z - 3) = 0$
- $8(x - 4) - 7(y - 7) - 6(z - 3) = 0$
- none of above

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NO CALCULATOR ALLOWED FOR THIS PROBLEM

7. For this problem we consider the point $P = (4, -1)$ and the function

$$f(x, y) = 20\sqrt{x} - 3y^2.$$

SHOW YOUR WORK TO RECEIVE FULL OR PARTIAL CREDIT!

(a) Find the gradient of f at P .

- $\langle 5, 6 \rangle$ $\langle 5, -6 \rangle$ $\langle 10, -6 \rangle$ $\langle 10, 6 \rangle$ none of above; correct is _____

(b) Find the maximum rate of change of f at P .

- $\sqrt{37}$ $\sqrt{47}$ $\sqrt{51}$ $\sqrt{61}$ $\sqrt{136}$

(c) Find the directional derivative of f at P in the direction from $P = (4, -1)$ to $Q = (1, 3)$.

- $\frac{39}{5}$ 39 $\frac{9}{5}$ 9 0 $-\frac{3}{\sqrt{13}}$ none of above; correct is _____

NO CALCULATOR ALLOWED FOR THIS PROBLEM

8. A farmer will install fences to form three adjacent rectangular pens next to a straight river, as shown. Each pen has dimensions x feet by y feet. No fencing is required along the river.

In this problem you will use Lagrange multipliers to determine maximum total area the farmer can enclose if he has a total of 120 feet of fence.



- (a) Express the total area enclosed in terms of x and y .

$$A(x, y) = \underline{\hspace{4cm}}$$

- (b) What is the constraint?

$3x + 4y = 120$
 $3x + y = 120$
 $x + 4y = 120$
 $x + \frac{y}{4} = 120$

- (c) List the three equations in three unknowns that arise from Lagrange multipliers.

- (d) Solve the system to determine the maximum total area the farmer can enclose.

WRITE YOUR ANSWER IN THE BLANK. SHOW YOUR WORK FOR FULL OR PARTIAL CREDIT.

Answer: $A =$