SM 122 **Test #2 Integration** 21 Feb 2007 (TSM)

Last Name: _____ Alpha: _____ SECTION: 1ST 2ND [CIRCLE ONE]

Instructions

1. **Do NOT turn the page or begin until instructed to do so.**
2. Failure to follow these instructions may result in points being deducted from your score.
3. Print your name, alpha, and circle your section above. Write nothing else on this cover page, except your signature on the line below to indicate you've read and understood the directions.
4. Point values for the problems are in the table below.
5. For fill-in-the-blank, multiple-choice, matching, and similar problems, write your answer directly on the test paper. There is plenty of space on each page (and the back) for your work. Although your work will not be graded, you might receive part-credit based on how "good" your incorrect answer is.
6. If a problem requests you to show your work, use the space provided to receive credit.
7. **Calculators are not allowed for this test.**
8. Unless otherwise indicated, leave answer in exact form; don't approximate $\sqrt{2}$ as 1.41, for instance.

Signature _____

DO NOT WRITE ANYTHING ON THIS PAGE BELOW THIS LINE

Problem	Points	Score
1	40	
2	120	
3	80	
4	60	
Total	**300**	

Test 1	**300**	
Test 2	**300**	
Quizzes (8 best)×5	**400**	
Total so far	**1000**	
A	800	80%
B	700	70%
C	600	60%
D	500	60%
F	< 500	$< 50\%$

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1. (a) Complete the statement of the formula for integration by parts.

$$\int u\,dv \quad = \quad \underline{\hspace{6cm}}$$

(b) Our proof in class of the integration by parts formula relied on:

Circle ALL correct capital letters.

(A) Product Rule for derivatives
(B) definition of a definite integral
(C) Fundamental Theorem of Calculus
(D) definition of derivative
(E) Chain Rule

(c) We may evaluate the integral

$$\int_0^1 2x \arctan(x)\,dx$$

using integration by parts with $v = x^2 + 1$.
This is an unusual choice for v, but it will work!
Fill in the three remaining blanks.

$u \quad = \quad \underline{\hspace{3cm}}$ $\qquad dv \quad = \quad \underline{\hspace{3cm}}$

$du \quad = \quad \underline{\hspace{3cm}}$ $\qquad v \quad = \quad \underline{\;x^2 + 1\;}$

(d) The value of the integral in (c) is

(A) $\pi/4$
(B) $\pi/2$
(C) $\pi/2 \; - \; 1$
(D) $\pi/4 \; + \; 1$
(E) $\pi + 1$

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2. Circle the best CAPITAL LETTER (not the entire answer) for each problem.

(a) What is the best way to evaluate the integral

$$\int_1^e \frac{\ln(x)}{x}\,dx\,?$$

(A) integrate by parts
(B) make a trigonometric substitution
(C) make a non-trigonometric substitution
(D) use partial fractions
(E) use the definition of the definite integral

(b) Suppose the continuous functions f and g satisfy $0 \le f(x) \le g(x)$ for all x. Circle ALL correct statements.

(A) If $\int_0^\infty f(x)\,dx$ converges, then $\int_0^\infty g(x)\,dx$ also converges.
(B) If $\int_0^\infty f(x)\,dx$ converges, then $\int_0^\infty g(x)\,dx$ diverges.
(C) If $\int_0^\infty f(x)\,dx$ diverges, then $\int_0^\infty g(x)\,dx$ also diverges.
(D) If $\int_0^\infty f(x)\,dx$ diverges, then $\int_0^\infty g(x)\,dx$ converges.
(E) None of the above statements is correct.

(c) If $x = \sec(\theta)$, then $\sin(\theta) =$ HINT: DRAW A RIGHT TRIANGLE

(A) $\sqrt{1-x^2}$
(B) $\sqrt{x^2-1}$
(C) $\sqrt{x^2-1}/x$
(D) $x/\sqrt{x^2-1}$
(E) $1/\sqrt{x^2-1}$

(d) Find the value of the constant A in the partial fraction expansion

$$\frac{8x+7}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

(A) $A = 0$
(B) $A = 1$
(C) $A = 3$
(D) $A = 5$
(E) $A = -3$

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(e) Each edge of a cubical aquarium has length 4 feet. The aquarium is filled halfway to the top with sea water, which has weight density δ lb/ft^3. What is the total hydrostatic force on the bottom surface of the aquarium?

HINT: Recall that weight density $= \delta = \rho g$.

(A) 2δ lb
(B) 4δ lb
(C) 8δ lb
(D) 16δ lb
(E) 32δ lb

(f) The improper integral

$$\int_0^\infty \frac{2}{1+x^2}\,dx$$

(A) diverges
(B) converges to $\pi/4$
(C) converges to $\pi/2$
(D) converges to π
(E) converges to 2π

(g) If the continuous function g satisfies $g(2) = 5$, $g(0) = 4$, and $g'(2) = 3$, then

$$\int_0^2 x \cdot g''(x)\,dx \quad =$$

(A) 1
(B) 5
(C) 6
(D) 7
(E) 9

(h) Evaluate the integral.

$$\int_0^{\pi/4} \cos^2(2\theta)\ d\theta$$

(A) π
(B) $\pi/2$
(C) $\pi/4$
(D) $\pi/8$
(E) $3\pi/8$

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3. Circle the best CAPITAL LETTER (not the entire answer) for each problem.

(a) Find the values of the constants $\boxed{h}$ and $\boxed{k}$ in the recurrence formula:

$$\int x^n e^{2x}\,dx \quad = \quad \boxed{\text{h}} \cdot x^n e^{2x} \quad - \quad \boxed{\text{k}} \int x^{n-1} e^{2x}\,dx$$

CIRCLE ONE CAPITAL LETTER FROM EACH COLUMN

(A)	$h = 1/n$	(A)	$k = n/2$
(B)	$h = 1/2$	(B)	$k = 2n$
(C)	$h = 1$	(C)	$k = 1/(n+1)$
(D)	$h = 2$	(D)	$k = 1/(2n+2)$
(E)	$h = n$	(E)	$k = 1/(2n)$

(b) After an appropriate trig substitution

$$\int_0^1 \frac{y^3}{\sqrt{4-y^2}}\,dy \quad =$$

(A) $8 \int_0^{\pi/6} \sin^2(\theta)\cos(\theta)\,d\theta$
(B) $16 \int_0^{\pi/6} \sin^3(\theta)\cos(\theta)\,d\theta$
(C) $8 \int_0^{\pi/6} \sin^3(\theta)\,d\theta$
(D) $4 \int_0^{\pi/6} [\sin^3(\theta)/\cos(\theta)]\,d\theta$
(E) none of above

(c) Evaluate the integral.

$$\int_0^{\pi/6} \sin^3(3\theta)\,d\theta$$

(A) 2/9
(B) 2/3
(C) 1
(D) 2
(E) 4

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(d) A rectangular lamina has vertices $(0,0)$, $(0,4)$, $(2,4)$, and $(2,0)$.
The constant density is $\rho = 8$ gm/unit^2.
Find the moment about the x-axis for the lamina.

HINT: WHAT IS THE MASS? WHAT IS $\overline{y}$?

(A) $M_x = 8$
(B) $M_x = 16$
(C) $M_x = 32$
(D) $M_x = 64$
(E) $M_x = 128$

(e) A lamina has constant density ρ gm/cm^2 and is bounded above by $y = f(x) = 1/x^3$, below by the positive x-axis, and on the left by the vertical line $x = 1$; the lamina extends forever to the right. Thus the mass of the lamina is given by the improper integral

$$m = \rho \int_1^\infty \frac{1}{x^3}\, dx$$

Find a similar definite integrals for the moments about the axes.

CIRCLE ONE CAPITAL LETTER FROM EACH COLUMN

(A)	$M_y = \rho \int_1^\infty \frac{1}{x}\, dx$	(A)	$M_x = (\rho/2) \int_1^\infty \frac{1}{x^2}\, dx$
(B)	$M_y = \rho \int_1^\infty \frac{1}{x^2}\, dx$	(B)	$M_x = (\rho/2) \int_1^\infty \frac{1}{x^3}\, dx$
(C)	$M_y = \rho \int_1^\infty \frac{1}{x^3}\, dx$	(C)	$M_x = (\rho/2) \int_1^\infty \frac{1}{x^4}\, dx$
(D)	$M_y = \rho \int_1^\infty \frac{1}{x^4}\, dx$	(D)	$M_x = (\rho/2) \int_1^\infty \frac{1}{x^5}\, dx$
(E)	$M_y = \rho \int_1^\infty \frac{1}{x^5}\, dx$	(E)	$M_x = (\rho/2) \int_1^\infty \frac{1}{x^6}\, dx$

(f) The integral

$$\int_{-1}^{2} \frac{1}{x^2}\, dx$$

(A) is proper
(B) is improper and diverges
(C) is improper and converges to 1/2
(D) is improper and converges to 1
(E) is improper and converges to 3/2

4. Fill in the blanks.

(a) Integrate.

$$\int \sec^2(10t)\, dt \quad = \quad \underline{\hspace{6cm}}$$

$$\int \cos(\pi y)\, dy \quad = \quad \underline{\hspace{6cm}}$$

$$\int 10^x\, dx \quad = \quad \underline{\hspace{6cm}}$$

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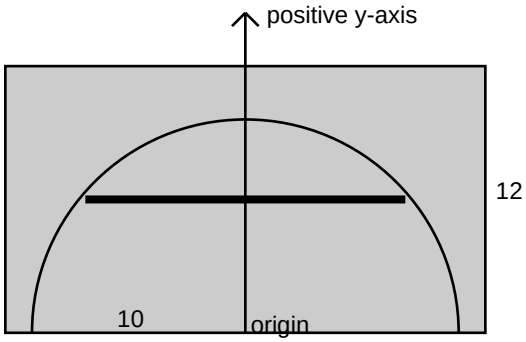
(b) Give the correct form of the partial fraction decomposition for $\frac{10x+9}{x^4+4x^2}$

DO NOT SOLVE FOR THE CONSTANTS A, B, ETC.

_____

(c) Give a definite integral for the hydrostatic force on the semi-circular disk with radius 10 m submerged in a tank that is 12 m deep, as shown. Express your integral in terms of g and ρ, the mass density of the liquid (in units of kg/m^3). The horizontal rectangle at position y is included to help you. Do not simplify or evaluate.

Hydrostatic force $= \int_{__}^{__} _____ \, dy$