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Riemann-Roch space representations from bad hyperelliptic curves: questions and computations

Computational Algebraic Geometry Conference
Idaho

5-27-2005

1 Introduction
- Motivation
- Borne's character formula
- Our multiplicity formula

2 Repns of big groups on R-R spaces of bad curves
- A bad hyperelliptic curve with a big automorphism group
- G-equivariant divisors and their RR spaces
- A really cool example

The main results described in this talk are the result of joint work with Amy Ksir. Some of this is "work in progress".

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$F = \mathbb{C}$ (for now)

C a non-singular projective curve over F, genus > 1, automorphism group G.

G acts on space of differentials on C and, more generally, on the Riemann-Roch space $L(D)$ (D G-equivariant divisor)

Questions: What are these representations?

Can we compute their character?

Their multiplicities?

Special cases: Eichler trace formula, Weil-Chevalley formula

General (in the extreme): the Borne character formula.

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F "arbitrary", $P \in C(F)$

G_P the subgroup of G fixing P

G_P acts on the cotangent space at P by an F-character. This is the **ramification character** of C at P.

ramification module is
$\Gamma_G = \sum_{P \in C(F)_{ram}} Ind_{G_P}^G (\sum_{\ell=1}^{e_P - 1} \ell \psi_P^\ell)$, where $e_P = |G_P|$, $\psi_P =$ramification character at P.

There is a unique G-module $\tilde{\Gamma}_G$ such that $\Gamma_G = |G| \tilde{\Gamma}_G$. By abuse of terminology call $\tilde{\Gamma}_G$ the **ramification module**

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reduced orbit: $D = \frac{1}{e_P} \sum_{g \in G} g(P)$, some $P \in C(F)$

The reduced orbits generate $Div(C)^G$.

equivariant degree: a map $deg_{eq} : Div(C)^G \to R(G)$, satisfying explicit conditions which can be computed if D is a reduced orbit.

Borne's formula: D a G-equivariant nonspecial divisor $\Longrightarrow$ (virtual) character of $L(D)$ is

$$[L(D)] = (1 - g(C/G))[F[G]] + [deg_{eq}(D)] - [\tilde{\Gamma}_G],$$

where $g(C/G)$ =genus of C/G, $\deg_{eq}(D)$ =equivariant degree of D, $\tilde{\Gamma}_G$ =ramification module.

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For each conjugacy class $\gamma_i \in G_*$, pick a cyclic subgroup H_i generated by a $g \in \gamma_i$.

Let $n_{\rho,j} = n_{\rho,j}(H) \geq 0$ denote the multiplicities $\langle \rho|_H, \psi^j \rangle_H$, (so $Res_H(\rho) = \sum_{j=1}^{|H|} n_{\rho,j}\psi^j$), where

$\rho =$irreducible representation of G,
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$\mathrm{supp}(D) = \cup_{i=1}^{s}\{g(P_i) \mid g \in G/G_{P_i}\}$, a disjoint union

write $\mathrm{supp}(D)/G = \{P_i \mid 1 \leq i \leq s\}$,

$$D = \sum_{P \in \mathrm{supp}(D)/G} r_P \sum_{g \in G/G_P} g(P),$$

for some $r_P \in \mathbb{Z}$.

Let $L(D)_\rho =$ the ρ-isotypical component of $L(D)$

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Let $G \subset Aut(C)$, D a G-equivariant nonspecial divisor.

Assume all irreducible characters of G are rational-valued.

multiplicity formula:

$$\dim(L(D)_\rho) = \dim(\rho)(1 - g(C/G)) + \sum_{P \in \mathrm{supp}(D)/G} T_{P,\rho}$$

$$- \sum_\ell (\dim(\rho) - \dim(\rho^{H_\ell})) \frac{R_\ell}{2},$$

(in Joyner-Ksir, "Decomposing representations of finite groups on Riemann-Roch spaces"),

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$$T_{P,\rho} = \begin{cases} \sum_{\ell=1}^{r_P} n_{\rho,\ell}(G_P) = \sum_{\ell=1}^{r_P} \langle \psi_P^{\ell}, \mathrm{Res}_{H_\ell}^{G} \rho \rangle, & r_P > 0, \\ -\sum_{\ell=0}^{-(r_P+1)} n_{\rho,-\ell}(G_P) = \sum_{\ell=1}^{-(r_P+1)} \langle \psi_P^{-\ell}, \mathrm{Res}_{H_\ell}^{G} \rho \rangle, & \\ \qquad r_P < 0, & \end{cases}$$

ρ^{H_ℓ} =restriction of ρ to H_ℓ,
$R_\ell = R(H_\ell)$ =number of branch points with decomposition group conjugate to H_ℓ,
ρ_1, ..., ρ_k a complete set of irreducible $\mathbb{Q}[G]$-modules (=irreducible $\mathbb{C}[G]$-modules).

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If C is a curve defined over a field F, $G = Aut_F(C)$ is finite with $|G| > |C(F)|$, then we call G **big**. If in addition char(F) divides $|G|$ then we call C **bad**.

Lemma: If G is large then every point of $C(F)$ is ramified for the covering $C \to C/G$.

Proof: Suppose $P \in C(F)$ is not ramified, so the stabilizer of P, G_P, is trivial. In this case, $|G \cdot P| = |G|/|G_P| = |G|$. But $G \cdot P \subset C(F)$ so $|G \cdot P| \leq |C(F)|$, a contradiction. QED

Open problem: Find multiplicity formulas for bad curves.

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$p \geq 5$ be a prime, $F = GF(p)$, $C : y^2 = x^p - x$

the weighted projective model: $Y^2 = X^p Z - XZ^p$ (here (X, Y, Z) $(x = X/Z, y = Y/Z^{g+1})$ with weights 1, $\frac{p+1}{2}$, and 1)

automorphism group $G = \mathrm{Aut}_F(C)$ is a central 2-fold cover of $G_0 = PSL(2, p)$, we have a short exact sequence,

$$1 \to Z \to G \to G_0 \to 1,$$

$$G \cong SL(2, p), \quad |SL(2, p)| = \frac{(p^2 - 1)(p^2 - p)}{p - 1}$$

where Z denotes the center of G (Z is generated by the hyperelliptic involution).

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G is generated by:

$$\gamma_1 = \begin{cases} x \longmapsto x, \\ y \longmapsto -y, \end{cases}, \quad \gamma_2 = \gamma_2(a) = \begin{cases} x \longmapsto a^2 x, \\ y \longmapsto ay, \end{cases}$$

$$\gamma_3 = \begin{cases} x \longmapsto x+1, \\ y \longmapsto y, \end{cases}, \quad \gamma_4 = \begin{cases} x \longmapsto -1/x, \\ y \longmapsto y/x^{\frac{p+1}{2}}, \end{cases}$$

where $a \in F^\times$ is a primitive $(p-1)^{st}$ root of unity and $Z = \langle \gamma_1 \rangle$.

G acts transitively on $C(F)$,

every point in

$$C(F) = \{[1:0:0], [0:0:1], [1:0:1], ..., [p-1:0:1]\}$$

is ramified over the covering $C \to C/G$

each stabilizer $G_P = \mathrm{Stab}_G(P)$ is non-trivial, $P \in C(F)$.

Let $P_1 = [1:0:1]$, $H = G_{P_1}$. H is a solvable group of order $p(p-1)$ generated by γ_1, $\gamma_2(a)$ and γ_3 (think of H as a "Borel subgroup" of G)

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for each $m \geq 1$, the Riemann-Roch space of $D = mP_1$ has a basis: $x^i y^j, \quad 0 \leq i \leq p-1,\ j \geq 0,\ 2i + pj \leq m.$

The semisimplification ρ_{ss} of the representation ρ of H acting on $L(D)$ is the direct sum of one-dimensional representations of H.

$$\gamma_3 : \begin{pmatrix} 1 \\ x \\ \vdots \\ x^r y^s \end{pmatrix} \longmapsto \begin{pmatrix} 1 \\ x+1 \\ \vdots \\ (x+1)^r y^s \end{pmatrix} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & \vdots \\ \vdots & \ddots & & 0 \\ 0 & \dots & r & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ \vdots \\ x^r y^s \end{pmatrix},$$

where the non-zero terms in bottom row of the matrix representation of γ_3 consist of the binomial coefficients $\frac{r!}{(r-j)!j!}$, $0 \leq j \leq r$.

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Properties of the function field $K = F(C) = F(x, y)$

genus $g = \frac{p-1}{2}$

$[K : F(y)] = p$

As an F-vector space

$$F(x, y) = F(y) \oplus xF(y) \oplus \ldots x^{p-1}F(y).$$

$K/F(y)$ is Galois and

$$\begin{array}{ccc} Gal(K/F(y)) & = & F \\ \sigma & \mapsto & a \\ \sigma(x) = x + a & & \end{array}$$

Properties of the function field $K = F(C) = F(x, y)$

genus $g = \frac{p-1}{2}$

$[K : F(y)] = p$

As an F-vector space

$$F(x, y) = F(y) \oplus xF(y) \oplus \ldots x^{p-1}F(y).$$

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Let $D_F = \sum_{P \in C(F)} P$, so $\deg(D_F) = |C(F)| = p + 1 = 2g + 2$ and therefore rD_F is non-special for each $r \geq 1$.

$$\begin{aligned} \dim L(rD_F) &= \deg(rD_F) - g + 1 \\ &= r(p+1) - \tfrac{p+1}{2} + 1 \\ &= (2r-1)g + 2r + 1 \end{aligned}$$

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Answer: No! No!

Irred $F[G]$-modules: Denote the irred degree n G-module by V_n, $V_n = \{\sum_i a_i X^i Y^{n-i-1}\}$, $n = 1, 2, ..., p$.
(These are all of them.)

Therefore, no G-module of dimension $p+1$ can be irreducible.

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Let $W = \mathrm{Span}\{ \frac{yx^k}{x^p - x} \mid 0 \leq k \leq \frac{p+1}{2} \}$.

$\dim W = \frac{p+3}{2}$.

W remains invariant under the action of γ_1, $\gamma_2(a)$, and γ_3

$\gamma_4 : \frac{yx^k}{x^p - x} \longmapsto -\frac{yx^{\frac{p+1}{2} - k}}{x^p - x}$,

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Lemma: $L(D_F) = W \oplus \mathbf{1}$, as G-modules.

Proof: W is a G-module and by definition, $W \subset L(D_F)$. $L(D_F)$ contains the constant functions on C, so $W \oplus \mathbf{1} \subset L(D_F)$. Since D_F is non-special, Riemann-Roch $\Longrightarrow \dim L(D_F) = 1 + \dim W$. QED

Questions: Is there a similar theorem for $L(rD_F)$?

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$$= a^{-1}\frac{a^{p+3}-1}{a^2-1} = a^{-1}\frac{a^4-1}{a^2-1} = a + a^{-1},$$

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when $n \geq 1$.

When $n = 2$ this agrees with $\operatorname{tr} \rho(\gamma_2(a))$.

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There are 8 F-rational points:

$C(F) = \{P_1 = [1:0:0], P_2 = [0:0:1], P_3 = [1:0:1], ..., P_8 = [6:0:1]\}$.

$H = Stab(P_1, G) =$group of order 42 generated by γ_1, γ_2, γ_3

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For each $m \geq 1$, the Riemann-Roch space $L(mP_1)$ has a basis consisting of monomials,

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Let $D = 5P_1$, $S = C(F) - \{P_1\}$, and let

$$C_D(S) = \{(f(P_2), ..., f(P_8)) \mid f \in L(D)_F\}.$$

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H fixes D and preserves S, so it acts on $C_D(S)$ via

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P =permutation automorphism group of this code.

$$|P| = 42 .$$

Note for Douglas Adams fans: Recall the ultimate answer to the universe is 42 .

The (permutation) action of G on this code implies that there is a homomorphism $\psi : H_1 \to P$.

Questions: What is the kernel of this map? Is it trivial??

Answer: Big kernel - non-trivial!

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$$\gamma_1 \leftrightarrow (2,7)(3,6)(4,5) = g_1,$$
$$\gamma_2 \leftrightarrow (2,5,3)(4,6,7) = g_2,$$
$$\gamma_3 \leftrightarrow (1,2,...7) = g_3.$$

$N = \ker(\psi) =$non-abelian normal subgroup of $H \cong \langle g_1, g_2, g_3 \rangle$ of order 21, $N = \langle g_2, g_3 \rangle$

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$$\gamma_1 \leftrightarrow (2,7)(3,6)(4,5) = g_1,$$
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$$\gamma_3 \leftrightarrow (1,2,...7) = g_3.$$

$N = \ker(\psi) =$non-abelian normal subgroup of $H \cong \langle g_1, g_2, g_3 \rangle$ of order 21, $N = \langle g_2, g_3 \rangle$

Questions: What is the character of $\rho : H \to Aut_F(L(5P_1))$?

The character table (over $\mathbb{C}$) of N is

Class	1	2	3	4	5
Size	1	7	7	3	3
Order	1	3	3	7	7
p = 7	1	2	3	1	1
χ_1	1	1	1	1	1
χ_2	1	ω	$-1-\omega$	1	1
χ_3	1	$-1-\omega$	ω	1	1
χ_4	3	0	0	ζ	ζ^3
χ_5	3	0	0	ζ^3	ζ

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The (Brauer) character table (over F) of N is

χ_{1a}	1	1	1	1	1
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χ_{1c}	1	ω	ω^2	1	1

(last two conjuacy classes are irregular mod 7)

The character table of N $\Longrightarrow$

$$\mathrm{tr}\rho = \chi_{1a} + \chi_{1b} + \chi_{1c}.$$

Open Problem: Generalize this $p = 7$ example.

Discussion topic: Does 42 have cosmic significance?

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The end

Thank you!