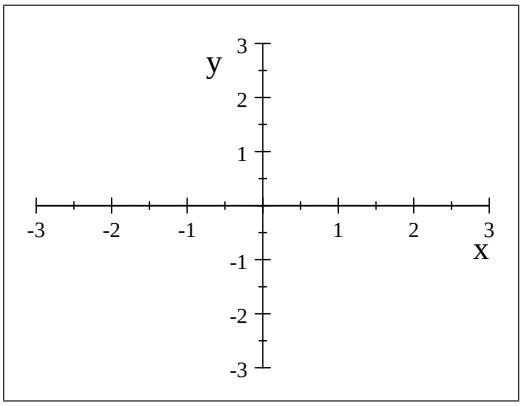
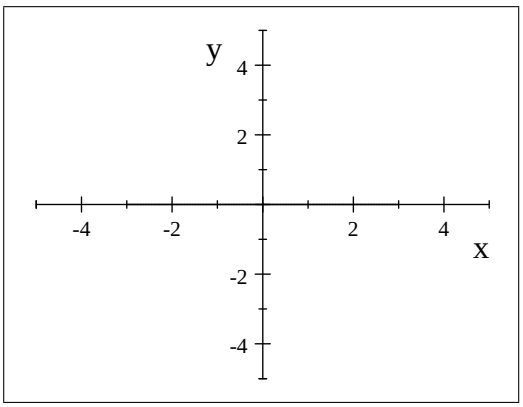
Mathematics SM122S - 2021 - Test #2 - 9 October 2008

PART 1 - No calculators allowed.

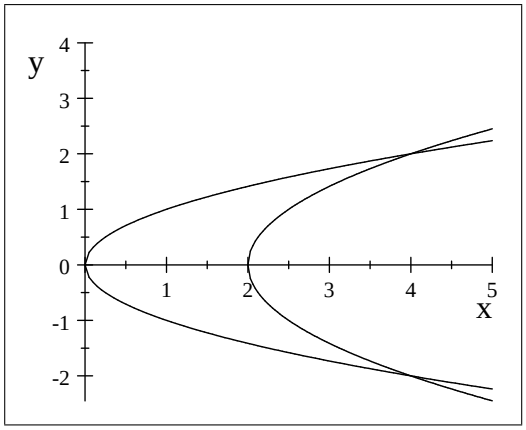
1. (11 points) Sketch the curve given by the parametric equations $\begin{cases} x = 2\cos(3t) \\ y = 3\sin(3t) \end{cases}$ $0 \leq t \leq \pi/2$.



2. (11 points) Sketch the curve given by the following equation in polar coordinates: $r = 4\sin\theta$



3. (30 points) Let R be the region bounded by $x = y^2$ and $x = \frac{1}{2}y^2 + 2$.



(a) Find the area of R.

(b) Find the volume of the solid obtained by revolving R about the $y-axis$.

(c) Find the volume of the solid obtained by revolving R about the horizontal line $y = 3$.

4. (12 points) A equilateral triangular plate of side length 2 feet is sitting vertically in a tank of water with one edge touching the surface of the water. Find the hydrostatic force on the plate. Recall that the density of water is 62.5 lbs/cu ft.

5. (12 points) A large block of ice is being lifted 25 feet. It is being lifted by a chain that is 25 feet long and weighs a total of 25 pounds. The ice weighs 100 pounds at the start and melts at a rate of 0.2 pounds per foot it is lifted. Find the work done in lifting the ice.

6. (12 points) A spherical tank of radius 3 meters is full of water. Find the work required to pump half the water to a point 2 meters above the top of the tank. Recall that the density of water is 1000 kg/m^3 and acceleration due to gravity is 9.8 m/sec^2.

7. (12 points) Find the area of the region inside $r = 2 - 2\cos\theta$ and outside $r = 1$.

8. **EXTRA CREDIT** A bucket is being lifted to the top of a tall building by a chain of length 50 feet at the rate of 2 feet per minute. The bucket, when empty, weighs 25 pounds. The chain weighs a total of 300 pounds. When the lifting begins, the bucket is filled with 100 pounds of acid. As the bucket is being lifted, the acid eats through the bucket causing the bucket to leak at the rate of $\sqrt[5]{t}$ pounds per minute where t is time in minutes since the lifting began. Find the work done in lifting the bucket.