NAME____

SM212 4021 PROF GAGLIONE, 6 APRIL 2009

TEST 3

Show ALL work in your blue book. Do NOT tear out any pages from blue book! Start each new problem on a new page.

1. (30 Points) Given $\frac{dx}{dt} = 4x + y$; $\frac{dy}{dt} = x + 4y$.

(a) Use e-val.'s and e-vec.'s to find the general solution. Your answer must be in the form $x = x(t)$ and $y = y(t)$, i.e., no matrices.

(b) Now consider $\frac{dx}{dt} = 4x + y + e^{-t}$; $\frac{dy}{dt} = x + 4y + e^{-t}$. Find its general solution.

2. (25 Points) Find all solutions, if there are any to the system

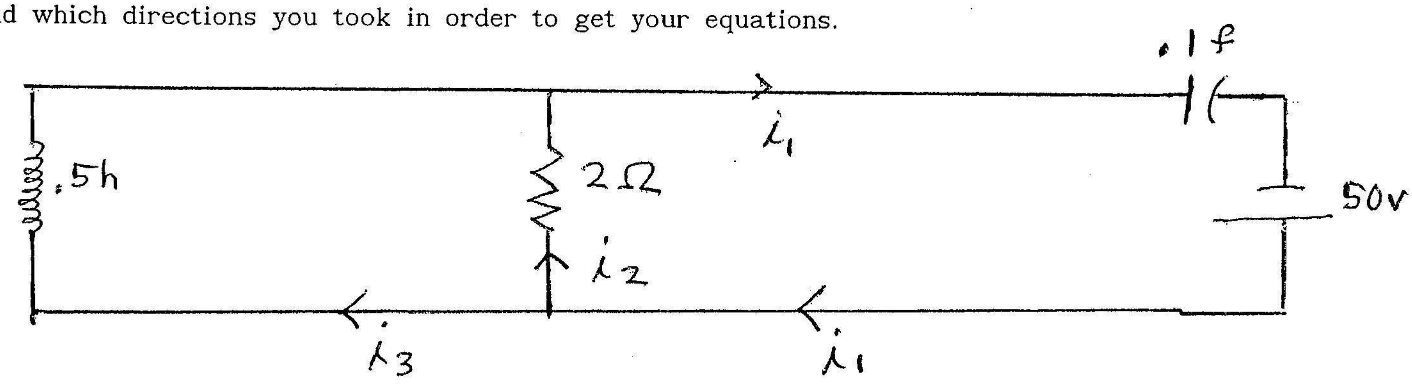
$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 4x_2 - x_3 + 2x_4 &= 11 \\ x_1 + x_2 + 2x_3 + 3x_4 &= 1 \end{aligned}$$

by row reducing to row reduced echelon form, BY HAND, the augmented matrix of the system. YOU MUST tell whether there are no solutions, exactly one solution or infinitely many.
If there is more than one solution, then give 3 distinct solutions .

3. (25 Points) (a) Use Euler's method for systems to find $x(2)$ and $y(2)$ with two steps $(n=2)$ in the system: **(you must do this by hand)**
$\frac{dx}{dt} = 4x + y$; $\frac{dy}{dt} = x + 4y$; $x(0) = -1$, $y(0) = 2$. Here you must explicitly state what the approximations for $x(2)$ and $y(2)$ are - just showing it in a table is NOT good enough!

(b) Now suppose A is a 3×3 matrix with an e-val -2 and corresponding e-vec $K = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. Find AK. (Here you can find AK even though you don't know what A is!)

4. (20 Points) Copy the following network into your blue book. Set up but do **NOT** solve 2 equations in i_2 and q_1 only. You must show on your schematic diagram which loops you took and which directions you took in order to get your equations.



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SM212 4021
TEST #3 SOLUTIONS

1. $\frac{dx}{dt} = 4x + y$

$\frac{dy}{dt} = x + 4y$

(a) $X = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow X' = \underbrace{\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}}_{A} X$

$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 - 1 =$

$16 - 8\lambda + \lambda^2 - 1 = \lambda^2 - 8\lambda + 15 = 0$

$(\lambda - 3)(\lambda - 5) = 0 \Rightarrow$

$\lambda_1 = 3,\ \lambda_2 = 5$

$\lambda_1 = 3$; So $K_1 = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$ is soln. to $(A - \lambda_1 I | 0) =$

$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{rref} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$ $k_1 + k_2 = 0$, $k_2 = k_2$, arb

$\Rightarrow$ $k_1 = -k_2$, $k_2 = k_2$, arb so $k_2 = 1 \Rightarrow k_1 = -1$

$\therefore \lambda_1 = 3;\ K_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_2 = 5$; So $K_2 = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$ is soln. to $(A - \lambda_2 I | 0) =$

$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right) \xrightarrow{rref} \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$ $k = k_2$, $k_2 = k_2$, arb

So $k_2 = 1 \Rightarrow k_1 = 1$

$\therefore \lambda_2 = 5,\ K_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\therefore$ g.s. $\Rightarrow$

$$X = c_1 e^{3t}\begin{pmatrix}-1\\1\end{pmatrix} + c_2 e^{5t}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}c_1(-e^{3t}) + c_2 e^{5t}\\ c_1 e^{3t} + c_2 e^{5t}\end{pmatrix} \Rightarrow$$

$$x = -c_1 e^{3t} + c_2 e^{5t}$$

$$y = c_1 e^{3t} + c_2 e^{5t}$$

(b)

$$\frac{dx}{dt} = 4x + y + e^{-t}$$

$$\frac{dy}{dt} = x + 4y + e^{-t}$$

$$\Rightarrow X' = AX + \begin{pmatrix}e^{-t}\\e^{-t}\end{pmatrix}$$

so this is nonhomo. with $F = \begin{pmatrix}e^{-t}\\e^{-t}\end{pmatrix}$

$$\Phi(t) = \begin{pmatrix}-e^{3t} & e^{5t}\\ e^{3t} & e^{5t}\end{pmatrix} \Rightarrow$$

$$X_p = \Phi \int \Phi^{-1} * F\,dt = \begin{pmatrix}-\frac{e^{-t}}{6}\\ -\frac{e^{-t}}{6}\end{pmatrix} \Rightarrow$$

g.s. $X = X_c + X_p$

$$= c_1\begin{pmatrix}-e^{3t}\\e^{3t}\end{pmatrix} + c_2\begin{pmatrix}e^{5t}\\e^{5t}\end{pmatrix} + \begin{pmatrix}-\frac{e^{-t}}{6}\\ -\frac{e^{-t}}{6}\end{pmatrix}$$

p 2

\# 2

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 4 \\ 2 & 4 & -1 & 2 & 11 \\ 1 & 1 & 2 & 3 & 1 \end{array}\right) \xrightarrow[-R_1+R_2]{-2R_1+R_2}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 4 \\ 0 & 0 & -3 & 0 & 3 \\ 0 & -1 & 1 & 2 & -3 \end{array}\right) \xrightarrow[-\frac{1}{3}R_2]{2R_3+R_1} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 5 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 2 & -3 \end{array}\right)$$

$$\xrightarrow[-R_2]{R_{23}} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 5 & -2 \\ 0 & 1 & -1 & -2 & 3 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right) \xrightarrow[R_3+R_2]{-3R_3+R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 1 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right)$$

$\Rightarrow$ $x_1 + 5x_4 = 1$

$x_2 - 2x_4 = 2$

$x_3 = -1$

$x_4 = x_4$, arb

$\Rightarrow$ all soln's are

$x_1 = 1 - 5x_4$

$x_2 = 2 + 2x_4$

$x_3 = -1$

$x_4 = x_4$, arb.

$\therefore$ there are infinitely many soln's

3 are: $x_4 = 0 \Rightarrow x_1 = 1, x_2 = 2, x_3 = -1$

$x_4 = 1 \Rightarrow x_1 = -4, x_2 = 4, x_3 = -1$

$x_4 = -1 \Rightarrow x_1 = 6, x_2 = 0, x_3 = -1$

3. $\dfrac{dx}{dt} = 4x + y$

(a) $\dfrac{dy}{dt} = x + 4y$ $\qquad x(0) = -1$, $\quad y(0) = 2$

$t_0 = 0,\ x_0 = -1,\ y_0 = 2 \qquad h = \dfrac{2-0}{2} = 1$

$t_1 = 1,\ x_1 = -1 + 1\left[4(-1) + 2\right] = -1 - 2 = -3$

$y_1 = 2 + 1\left[(-1) + 4(2)\right] = 2 + 7 = 9$

$t_2 = 2,\ x_2 = -3 + 1\left[4(-3) + 9\right] = -3 - 3 = -6$

$y_2 = 9 + 1\left[-3 + 4(+9)\right] = 9 + 33 = 42$

$$\boxed{\begin{aligned} \therefore\ x(2) &\approx -6 \\ y(2) &\approx 42 \end{aligned}}$$

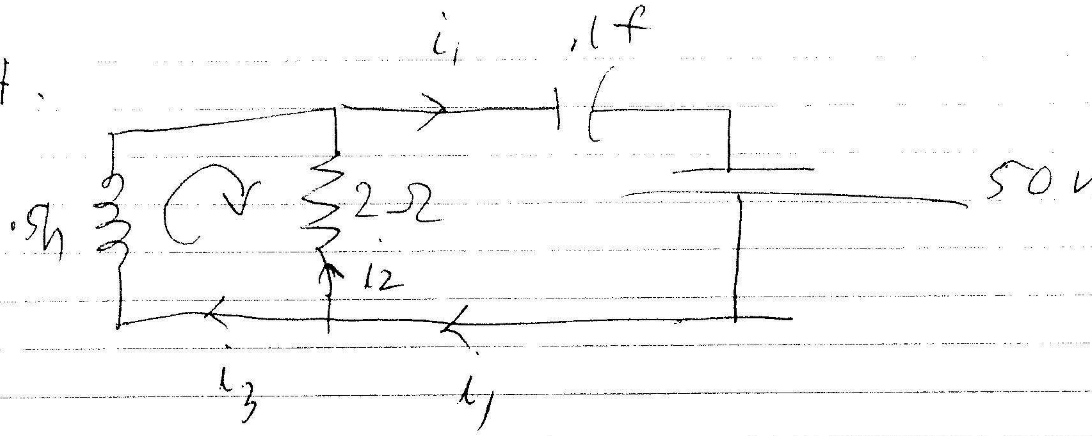
t	x	y
0	-1	2
1	-3	9
2	-6	42

(b) $AK = \lambda K = (-2)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$

t	x	$hf = 4x+y$	y	$hg = x+4y$
0	-1	-2	2	7
1	-3	-3	9	33
2	6		42	

P3

4.



Fr. KI $i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2 = q_1' - i_2$

LH Loop: $-2i_2 + .5\frac{di_3}{dt} = 0$

RH Loop: $\frac{1}{.1}q_1 + 2i_2 = 50 \Rightarrow$

$10q_1 + 2i_2 = 50$ (already in terms of q_1 & i_2)

Sub'ing in i_3, $\frac{di_3}{dt} = q_1'' - i_2'$

$\Rightarrow$ for LH Loop

$-2i_2 + \frac{1}{2}(q_1'' - i_2') = 0$

or $-4i_2 - i_2' + q_1'' = 0$ (1)

$i_2 + 5q_1 = 25$ (2)

(another possible eq.: $q_1'' + 20q_1 - i_2' = 100$)