Undetermined coefficients example

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Example Problem: Solve $y'' + y = \sin t + e^t$.

We solve this using SAGE. Here is the SAGE code used to solve this.

```
yh = maxima.de_solve('diff(y,t,2) + y = 0', ['t','y'])
maxima("de:lambda ([y], diff(y,t,2)+y-sin(t)-exp(t))")
eqn = maxima("de(A1*exp(t)+A2*t*cos(t)+A3*t*sin(t))")
eqn
```

which produces (1) below.

The general form of the particular solution in the method of undetermined coefficients is $y_p = A_1 e^t + A_2 t \cos t + A_3 t \sin t$ (since the homogeneous part of the solution is $y_h = c_1 \cos(t) + c_2 \sin(t)$, we must multiply the $\cos(t), \sin(t)$ terms by t). Substituting gives

$$(A_1 e^t + A_2 t \cos t + A_3 t \sin t)'' + (A_1 e^t + A_2 t \cos t + A_3 t \sin t) = \sin t + e^t.$$

and so we must solve for the undetermined coefficients A_1, A_2, A_3 in

$$2A_1 e^t - A_2 2 \sin t + 2A_3 \cos t = \sin t + e^t = 0 \tag{1}$$

We get $A_1 = 1/2$, $A_2 = -1/2$, $A_3 = 0$.

To check this, use the SAGE command

```
maxima.de_solve('diff(y,x,2) + 3*x = y', ['x','y'])
```

EXERCISES: Using SAGE, solve
(a) $y'' - y = \sin t + e^t$
(b) $y'' - 2y = e^t + te^{2t}$.

You can download SAGE from http://sage.sf.net. Installation instructions are at http://sage.sourceforge.net/documentation.html .