SM212 Final Examination: Multiple Choice Section

Practice Examination

1. The differential equation $\frac{dy}{dx} = x - y$ is

   a) separable b) linear c) homogeneous d) none of these

2. The differential equation $\frac{dy}{dx} = (x + y + 1)^2$ is

   a) separable b) linear c) homogeneous d) none of these

3. A 120 gallon tank initially contains 90 lb of salt dissolved in 90 gallons of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min and the well-stirred mixture flows out of the tank at the rate of 3 gal/min. A differential equation for the amount x (in pounds) of salt in the tank is

   a) $\frac{dx}{dt} + \frac{1}{30}x = 8$ b) $\frac{dx}{dt} + \frac{3}{90+2t}x = 4$

   c) $\frac{dx}{dt} + \frac{3}{90+t}x = 8$ d) $\frac{dx}{dt} + 90x = 2$

4. For the direction field shown in Figure 1 the Euler method approximation to $y(1)$ with an initial condition $y(0) = 1$ and a step size of $h = 1$ is

   a) 0 b) 1 c) 2 d) 3

5. For the initial value problem $y' = x + y, y(0) = 1$ the improved Euler method approximation to $y(0.1)$ with a step size of $h = 0.1$ is

   a) 2.21 b) 1.11 c) 1.00 d) 0.99

6. A solution to the differential equation $y''' + 3y'' + 3y' + y = 0$ is $y(t) =$

   a) $e^{-t} - t^2e^{-t}$ b) te^t c) $6\sin t$ d) $1 + te^t$

7. For an LRC series circuit with $L = 2$ henrys, $R = 14$ ohms, $C = 0.05$ farads, and an electromotive force of $E = 120$ volts, the steady-state charge on the capacitor is $q =$

   a) 120 b) 6 c) $60t$ d) $120t$

8. The appropriate guess for the particular solution to the differential equation $y'' + 2y' + y = 3 - 2\sin x$ is $y_p =$

   a) $A + B\sin x$ b) $Ax + Bx^2 + C\cos x + D\sin x$

   c) $A + Bx\cos + Cx\sin x$ d) $A + B\cos x + C\sin x$

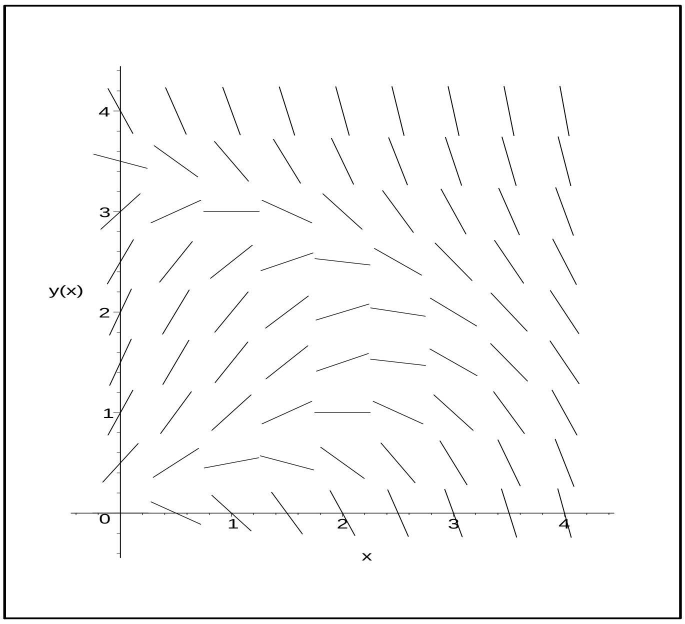

Figure 1: Direction field for Problem 4.

9. A mass weighing 100 lbs is attached to the end of a spring that is stretched 1 inch by a force of 50 lbs. An external force of $5\cos\omega t$ acts on the mass. There is no damping force present. At what frequency (in Hertz) will resonance occur? Assume that acceleration due to gravity at the earth's surface is $g = 32$ ft/sec^2.

   a) $\frac{4\sqrt{3}}{\pi}$ b) $8\sqrt{3}$ c) $\frac{1}{\pi}$ d) 2

   Problems 10 and 11 refer to Figure 2.

10. Figure 2 (top) shows the response of the amplitude of the steady-state solution to a damped mass-spring system subject to an external force $F_0 \cos\omega t$. The resonant driving angular frequency is approximately $\omega =$

    a) 0 b) 1 c) 3 d) 8

11. Figure 2 (bottom) shows the position $x(t)$ of the mass for the same mass-spring system as Figure 2 (top) when driven at a particular angular frequency. This driving angular frequency is approximately $\omega =$

    a) 2 b) 3 c) 4 d) 5

12. $\mathcal{L}\{t^{1.5}\} =$

    a) $\frac{2.5\Gamma}{s^{2.5}}$ b) $\frac{3\sqrt{\pi}}{4s^{2.5}}$ c) $\frac{6}{s^{2.5}}$ d) $\frac{1}{s^{2.5}}$

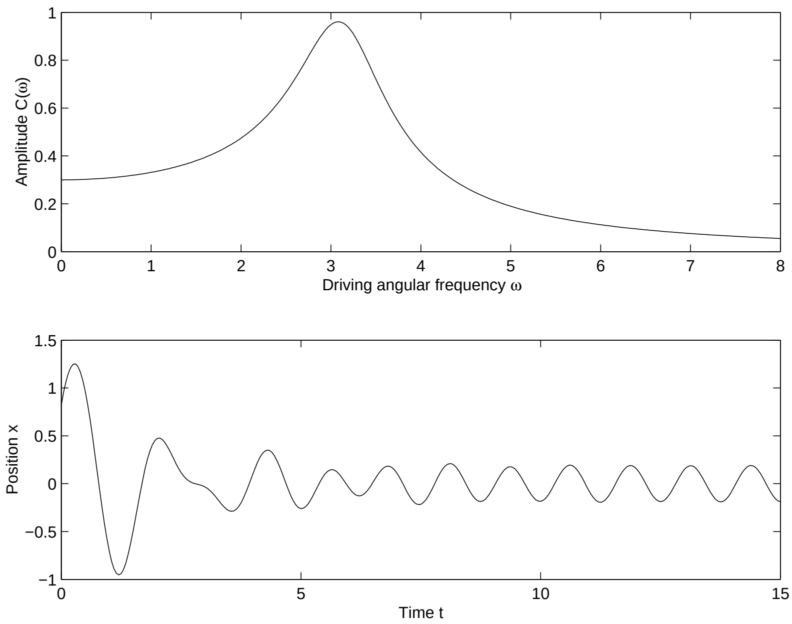

Figure 2: Graphs for Problems 10 and 11.

13. The Laplace transform of the function $f(t) = \begin{cases} t, 0 \le t \le 1 \\ 2-t, t > 1 \end{cases}$ is

   a) $\frac{1-2e^{-s}}{s^2}$ b) $\frac{2}{s}$ c) $\frac{1}{s^2} - \frac{2e^{-s}}{s}$ d) $\frac{1}{s^2} + 2e^{-s}\frac{1}{s} - \frac{1}{s^2}$

14. The Laplace transform of the periodic function given over one period by $f(t) = \begin{cases} 1, 0 < t < 1 \\ 0, 1 < t < 2 \end{cases}$ is

   a) $\frac{1-e^{-s}}{s}$ b) $\frac{1}{1-e^{-s}}$ c) $\frac{1}{1-e^{-2s}}\left(\frac{e^{-s}}{s} - \frac{1}{s}\right)$ d) $\frac{1}{s(1+e^{-s})}$

15. For the initial value problem $x'' + 4x' + 13x = t, x(0) = -1, x'(0) = 1$ the Laplace transform $X(s)$ of the solution $x(t)$ is

   a) $\frac{1}{s^2+4s}\left(-s-3-\frac{1}{s}+\frac{1}{s^2}\right)$ b) $\frac{1}{s^2+4s}\left(-s-\frac{1}{s}+\frac{1}{s^2}\right)$

   c) $-\frac{s+3}{s^2+4s+13} + \frac{1}{s^2(s^2+4s+13)}$ d) $-\frac{s}{s^2+4s+13} + \frac{1}{s^2(s^2+4s+13)}$

   Problems 16 and 17 refer to the two-mass, two-spring system shown in Figure 3. The differential equations describing this system are

$$m_1 x_1'' = -(k_1+k_2)x_1 + k_2 x_2 + F_0 \cos \omega t$$
$$m_2 x_2'' = k_2 x_1 - k_2 x_2$$

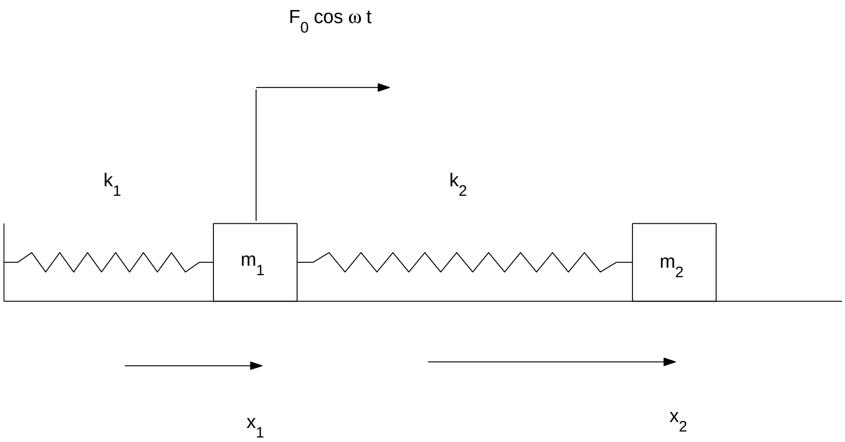

Figure 3: Mass-spring system for Problems 16 and 17.

16. When $m_1 = 6, m_2 = 1, k_1 = 36, k_2 = 6$ and no external force is applied $(F_1 = 0)$ the natural frequencies of vibration are

    a) $\omega_1 = 0, \omega_2 = 1$ b) $\omega_1 = -4, \omega_2 = -9$

    c) $\omega_1 = 2, \omega_2 = 3$ d) $\omega_1 = 2, \omega_2 = 4$

17. When $m_1 = 1, k_1 = 1, k_2 = 10, F_0 = 5, \omega = 10$ the system has a solution of the form $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \cos 10t$ for which $x_1 \equiv 0$ if $m_2 =$

    a) 0 b) 0.05 c) 0.1 d) 0.2

18. A periodic function is given over one period by $f(t) = |t|, -\pi < t < \pi$. Which of the following statements is correct for the Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}$ of $f(t)$

    a) $a_n = 0$ for all even integers n, but not for any odd integers n.

    b) $a_n = 0$ for all odd integers n, but not for any even integers n.

    c) $a_n = 0$ for $n = 0, 1, 2, \ldots$.

    d) $a_n = 0$ for $n = 1, 2, 3, \ldots$, but $a_0 \neq 0$.

19. At the point $t = 0$ the Fourier sine series of the function $f(t) = 1, -0 \leq t \leq 1$ converges to

    a) -1 b) 0 c) $\frac{1}{2}$ d) 1

20. A metal bar of length 2 and thermal diffusivity 3 has its left end held at $0°$ and its right end insulated for all times $t > 0$. Mathematically this is described by the partial differential equation and boundary conditions

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}, u(0,t) = 0, \frac{\partial u}{\partial x}(2,t) = 0$$

The physically reasonable solutions of the form $u(x,t) = X(x)T(t)$ which satisfy the partial differential equation are $u(x.t) = e^{-3\lambda^2 t}(A\cos\lambda x + B\sin\lambda x)$. The solutions of this form which also satisfy the boundary conditions are $u(x,t) =$

a) $e^{-3\left(\frac{n\pi}{2}\right)^2 t} B \sin\frac{n\pi x}{2}, n = 1,2,3,\ldots$ b) $e^{-3\left(\frac{n\pi}{2}\right)^2 t} A \cos\frac{n\pi x}{2}, n = 1,2,3,\ldots$

c) $e^{-3\left(\frac{n\pi}{4}\right)^2 t} B \sin\frac{n\pi x}{4}, n = 1,3,5,\ldots$ d) $e^{-3\left(\frac{n\pi}{4}\right)^2 t} A \cos\frac{n\pi x}{4}, n = 1,3,5,\ldots$

Answers: 1. b, 2. d, 3. c, 4. d, 5. b, 6. a, 7. b, 8. d, 9. a, 10. c, 11. d, 12. b, 13. a, 14. d, 15. c, 16.c, 17. c, 18. a, 19. b, 20. c