SM212, Prof Joyner, 2-27-2004
Practice Test 2

PROBLEM 1: Find the general form of y_p. Do not solve for the undetermined coefficients. Do not solve the ODE.
(a) $y'' - 2y' + y = xe^x$,
(b) $y'' - 3y' + 2y = e^x + e^{2x} + \sin(x)$.

(Ans: (a) $y_p = A_1x^3e^x + A_2x^2e^x + A_3xe^x$, (b) $y_p = A_1xe^x + A_2xe^{2x} + A_3\cos(x) + A_4\sin(x)$.)

PROBLEM 2: An RLC circuit has a 1 henry inductor in series with a 4 ohm resistor and a 1/5 farad capacitor. A battery of $\sin(t)$ volts is attached.

(a) Assume that the initial charge and curent is zero. Find the current at time t.

(b) Identify the transient and steady state terms.

(Ans: (a) $q(t) = \frac{1}{8}\exp(-2t)\cos(t) + \frac{1}{8}\exp(-2t)\sin(t) - \frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$, $i(t) = -\frac{1}{8}\exp(-2t)\cos(t) - \frac{3}{8}\exp(-2t)\sin(t) + \frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$, (b) transient: $\frac{1}{8}\exp(-2t)\cos(t) + \frac{1}{8}\exp(-2t)\sin(t)$, steady state: $-\frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$)

PROBLEM 3: Solve

$$x'' + 4x = \tan(2t),\ x(0) = 0,\ x'(0) = 0.$$

(Ans: $x(t) = \frac{1}{4}\sin(2t) - \frac{1}{4}\cos(2t)\ln\left(\frac{1+\sin(2t)}{\cos(2t)}\right)$)

PROBLEM 4: A 1 kg object is attached to a spring suspended from the ceiling. The spring has spring constant 4 and has an external force of $\sin(\gamma t)$ acting on it.

(a) Find the γ for which the displacement is in a state of pure resonance (i.e., has the same period as the oscillations of the homogeneous part of the solution).

(b) For this γ in (a), find the displacement if the mass is initially released with a 1 m/s downward velocity at equilibrium. (If you don't know the answer to (a), just take $\gamma = 1$.)

(Ans: (a) $\gamma = 2$, (b) $x(t) = -\frac{1}{4}t\cos(2t) + \frac{5}{8}\sin(2t)$)

PROBLEM 5: Solve $y'''' - y = e^x$.

(Ans: $y(x) = \frac{1}{4}x\exp(x) - \frac{3}{8}\exp(x) + c_1\exp(x) + c_2\sin(x) + c_3\cos(x) + c_4\exp(-x)$)

PROBLEM 6: An RLC circuit has a 1/2 henry inductor in series with a 3 ohm resistor and a 1/5 farad capacitor. A battery of $39\sin(t)$ volts is attached.

1. Assume that the initial charge and current is zero. Find the charge at time t.

2. Identify the transient and steady state terms.

(Ans: $6e^{-3t}\sin(t)+4e^{-3t}\cos(t)-4\cos(t)+6\sin(t)$, transient=$6e^{-3t}\sin(t)+4e^{-3t}\cos(t)$, steady state=$-4\cos(t)+6\sin(t)$)