Sm212, Review 2

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This is a set of review problems. It may not be sufficient preparation for the test, but it covers most basic topics.

1. Give the definition of the functions $f_1(x), f_2(x), \ldots, f_n(x)$ being linearly dependent on an interval I. What is a fundamental set of solutions of a homogeneous linear differential equation?

2. Test each set of functions for linear dependence or linear independence on $(-\infty, \infty)$.

$$\{\sin(3x), \cos(3x)\}, \qquad \left\{-2, \cos^2 x, 3\sin^2(x)\right\}, \quad \{e^x, xe^x\}$$

   Answer: The first set is linearly independent, the second linearly dependent, the third is linearly independent.

3. Find the general solution of each of the following DEs.

$$y'''+4y''-3y'-18y=0, \quad y'''-9y''+27y'-27y=0, \quad y''-2y'+10y=0$$

   Answer:

$$y=c_1e^{-3x}+c_2xe^{-3x}+c_3e^{2x}, \quad y=c_1e^{3x}+c_2xe^{3x}+c_3x^2e^{3x}, \quad y=c_1e^x\cos(3x)+c_2e^x\sin(3x)$$

4. Compute annihilators for each of the following expressions

$$x-2x^3+2xe^{-2x}-\sin 4x, \qquad xe^{3x}\cos(7x)-x^2\sin(x)$$

Answer:

$$D^4 (D+2)^2 \left(D^2+16\right), \qquad \left(D^2-6D+58\right)^2 \left(D^2+1\right)^3$$

5. Solve the following differential equation by using undetermined coefficients with annihilators.

$$\left(D^2-4\right) y = e^x + 2e^{2x}$$

Answer:

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{3} e^x + \frac{1}{2} x e^{2x}$$

6. Solve the following differential equation by using undetermined coefficients with annihilators.

$$y'' + y = e^x + \cos x$$

Answer:

$$c_1 \cos x + c_2 \sin x + \frac{1}{2} e^x + \frac{1}{2} x \sin x$$

7. Use variation of parameters to solve the differential equation.

$$y'' + y = \tan x$$

Answer:

$$y(x) = c_1 \cos(x) + c_2 \sin(x) - \cos x \ln|\sec x + \tan x|$$

8. A 2 lb weight stretches a vertical spring 6 inches. If the weight is released 8 inches below the equilibrium, with an upward velocity of 4/3 ft/s, compute $x(t)$, the amplitude, period, frequency, and phase angle.
Answer:

$$\frac{1}{16} x'' + 4x = 0, \qquad x'' + 64x = 0, \quad x(0) = \frac{2}{3}, \quad x'(0) = -\frac{4}{3}$$

solve to get

$$x(t) = \frac{2}{3}\cos(8t) - \frac{1}{6}\sin(8t) = \frac{\sqrt{17}}{6}\sin(8t + a)$$

where $a = 1.815\,8$ rad.

9. Solve each DE and say if it is underdamped, overdamped, or critically damped.

$$\begin{aligned} x'' + 5x' + 4x &= 0, \quad x(0) = x'(0) = 1 \\ x'' + 2x' + 10x &= 0, \quad x(0) = -2, \; x'(0) = 0 \end{aligned}$$

Answer:

$$x(t) = \frac{5}{3}e^{-t} - \frac{2}{3}e^{-4t}, \qquad x(t) = e^{-t}\left(-2\cos(3t) - \frac{2}{3}\sin(3t)\right)$$

The first is overdamped, the second is underdamped.

10. Find the value of c such that a spring-mass system of mass $m = 2$, spring constant $k = 8$ and external force $3\cos(ct)$ undergoes pure resonance.
Answer:

$$c = 2$$