Investigating Coding Theory Using Python

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BUSES
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110 NORTH
CARPOOLS
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Century Blvd
105 WEST
LAX Airport
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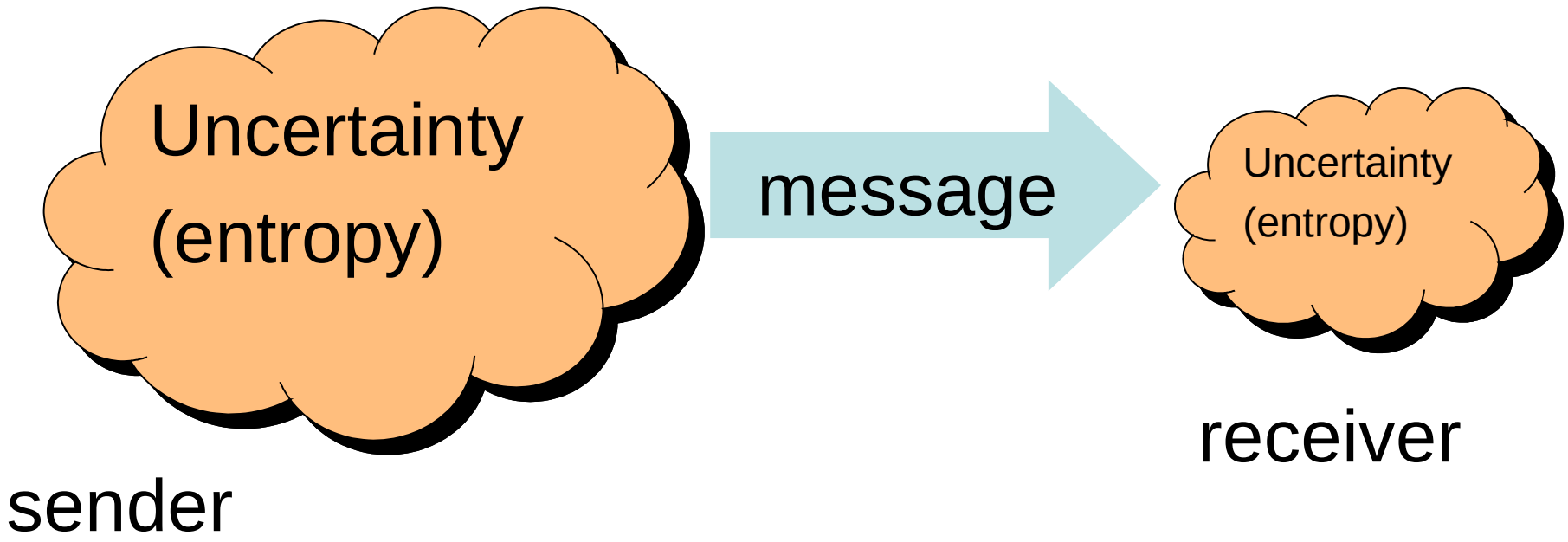
Half the job is finding the problem

The purpose of communication?



Information environments

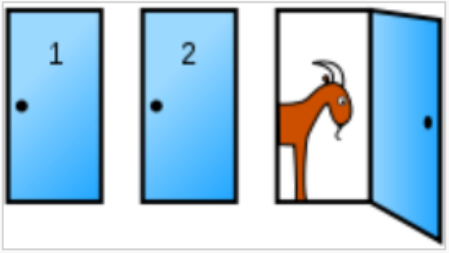
Monty Hall problem

From Wikipedia, the free encyclopedia

The **Monty Hall problem** is a probability puzzle based on the American television game show *Let's Make a Deal*. The name comes from the show's host, Monty Hall. The problem is also called the **Monty Hall paradox**, as it is a veridical paradox in that the result appears absurd but is demonstrably true.

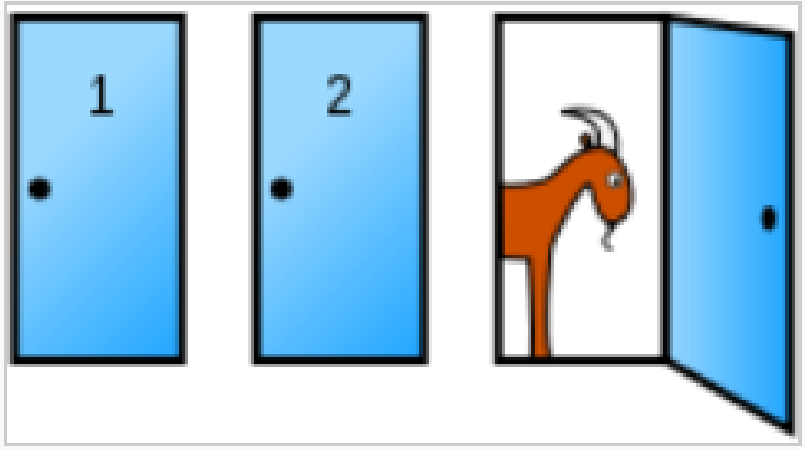
A well-known statement of the problem was published in *Parade* magazine:

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Whitaker 1990)



In search of a new car, the player picks a door, say 1. The game host then opens one of the other doors, say 3, to reveal a goat and offers to let the player pick door 2 instead of door 1.

How has the information environment changed?



In search of a new car, the player picks a door, say 1. The game host then opens one of the other doors, say 3, to reveal a goat and offers to let the player pick door 2 instead of door 1.

Should you change your guess or not?

Plot Summary for

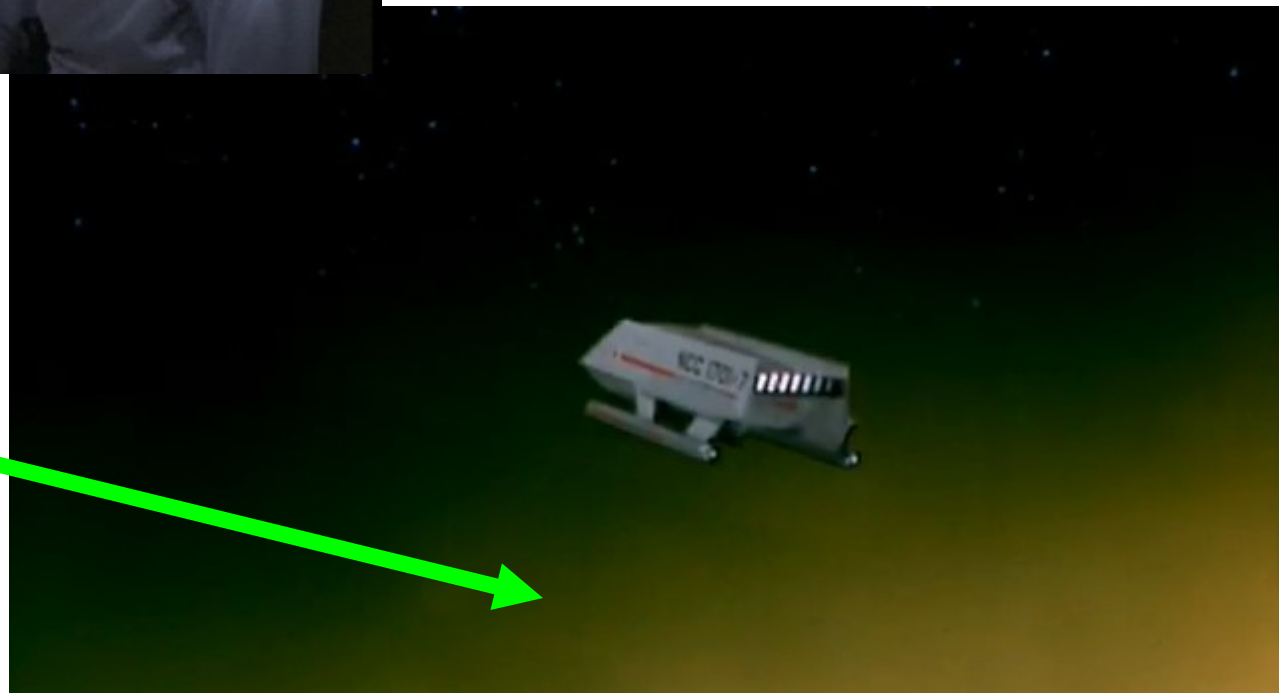
"Star Trek" The Galileo Seven (1967)

Prev | 17 of 80 Episodes | Next

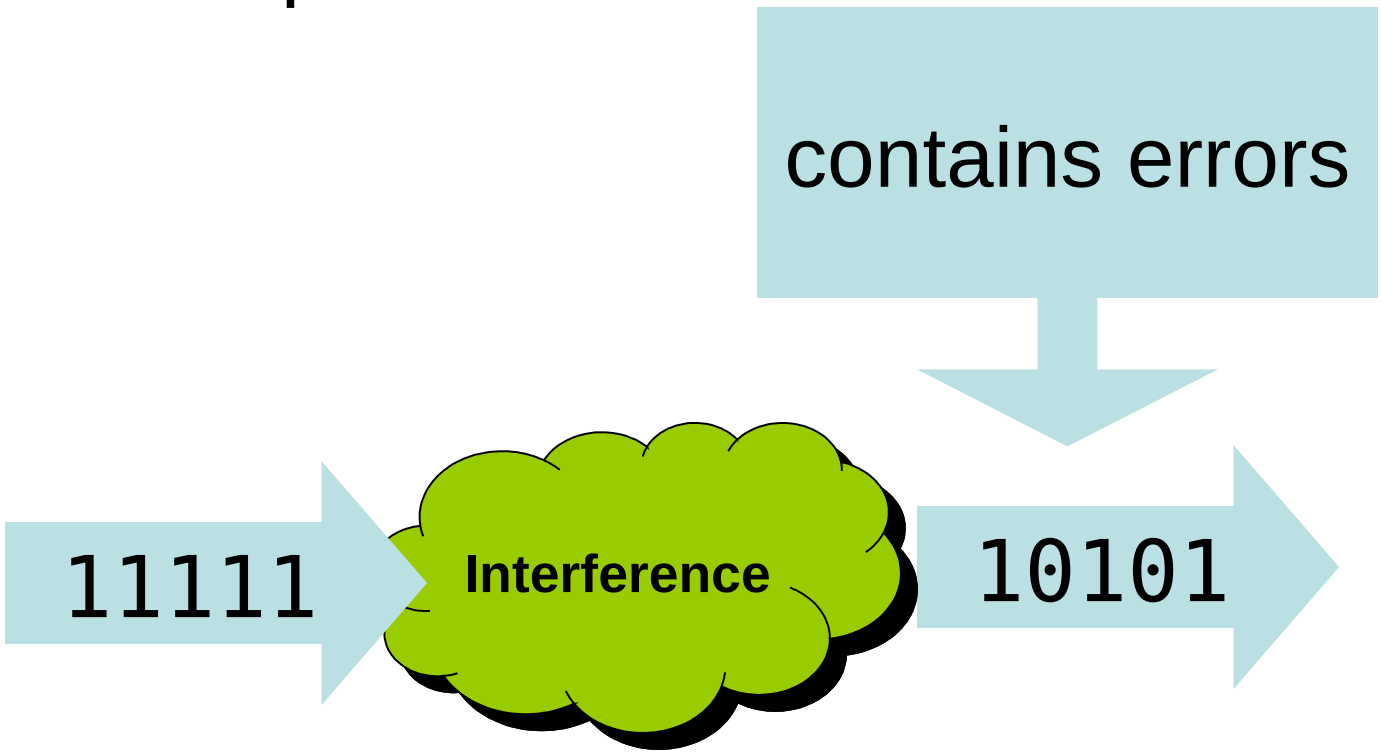
A shuttle craft under Mr. Spock's command is forced to land on a hostile planet. His emotionless approach to command does not sit well with some crew members, particularly Mr. Boma who challenges Spock at every opportunity. The Enterprise and Captain Kirk meanwhile have only a short time to find the lost shuttle craft as they must deliver urgent medical supplies to Markus III in only a few days. *Written by garykmcd*

What information must be sent?

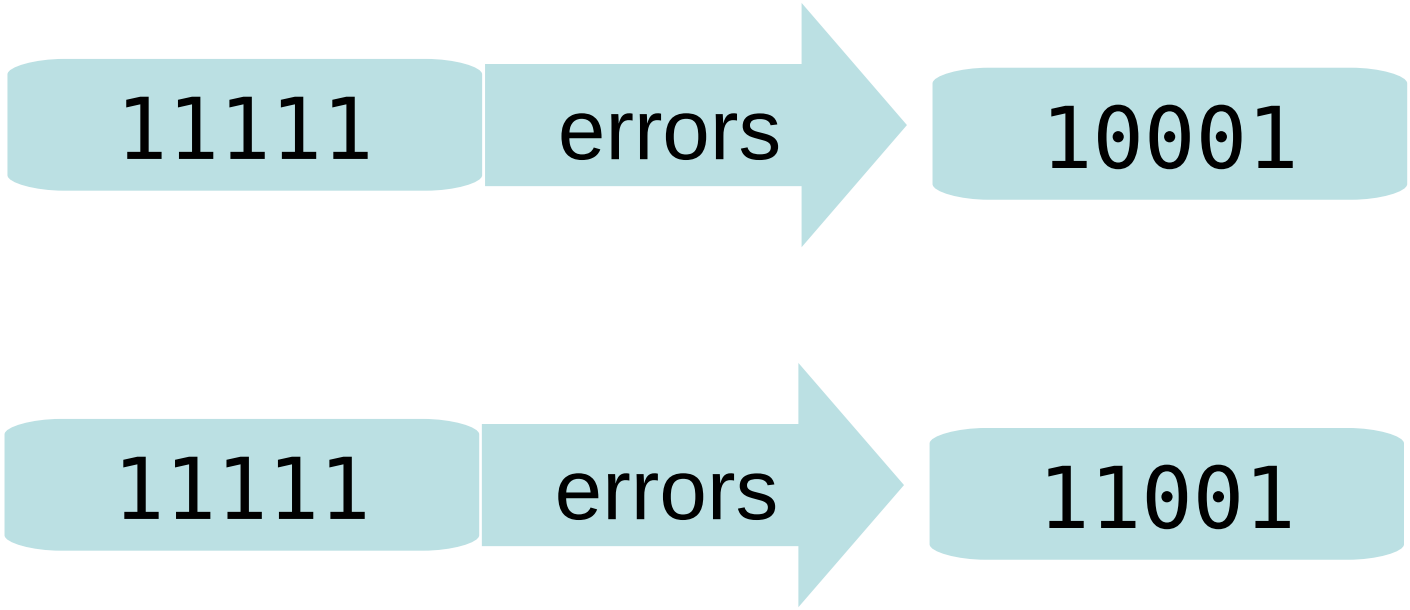
Interference environment



Repetition code

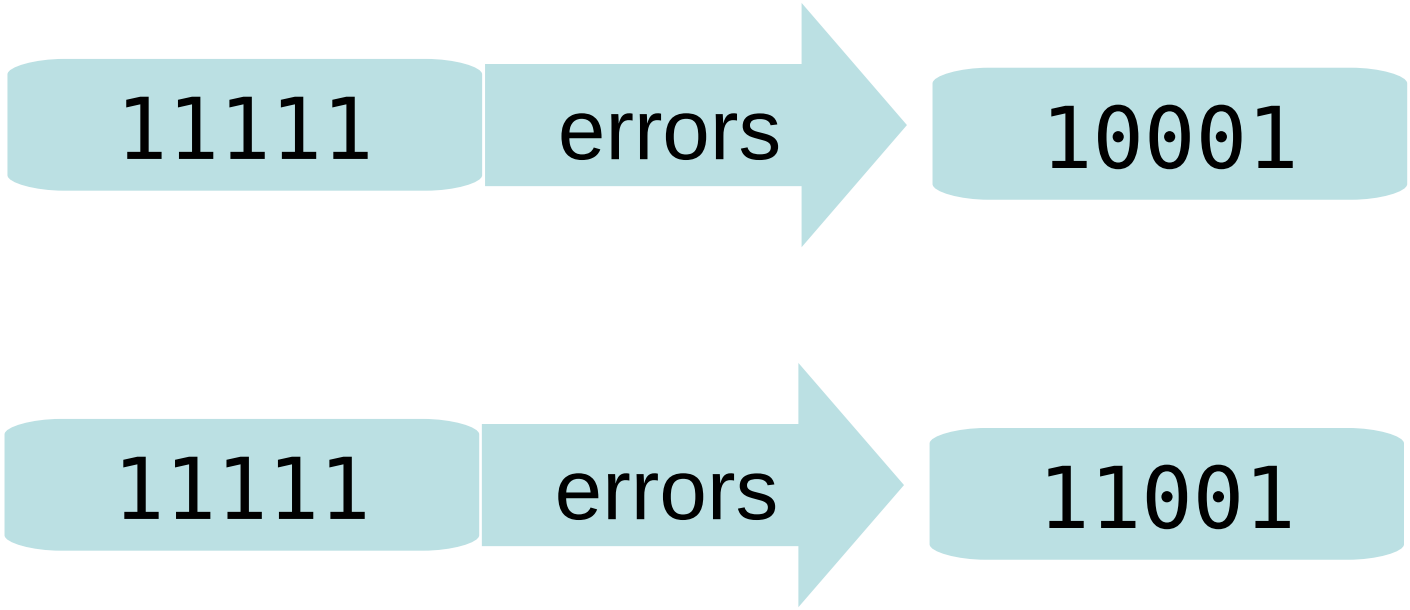


What errors can I recover from?



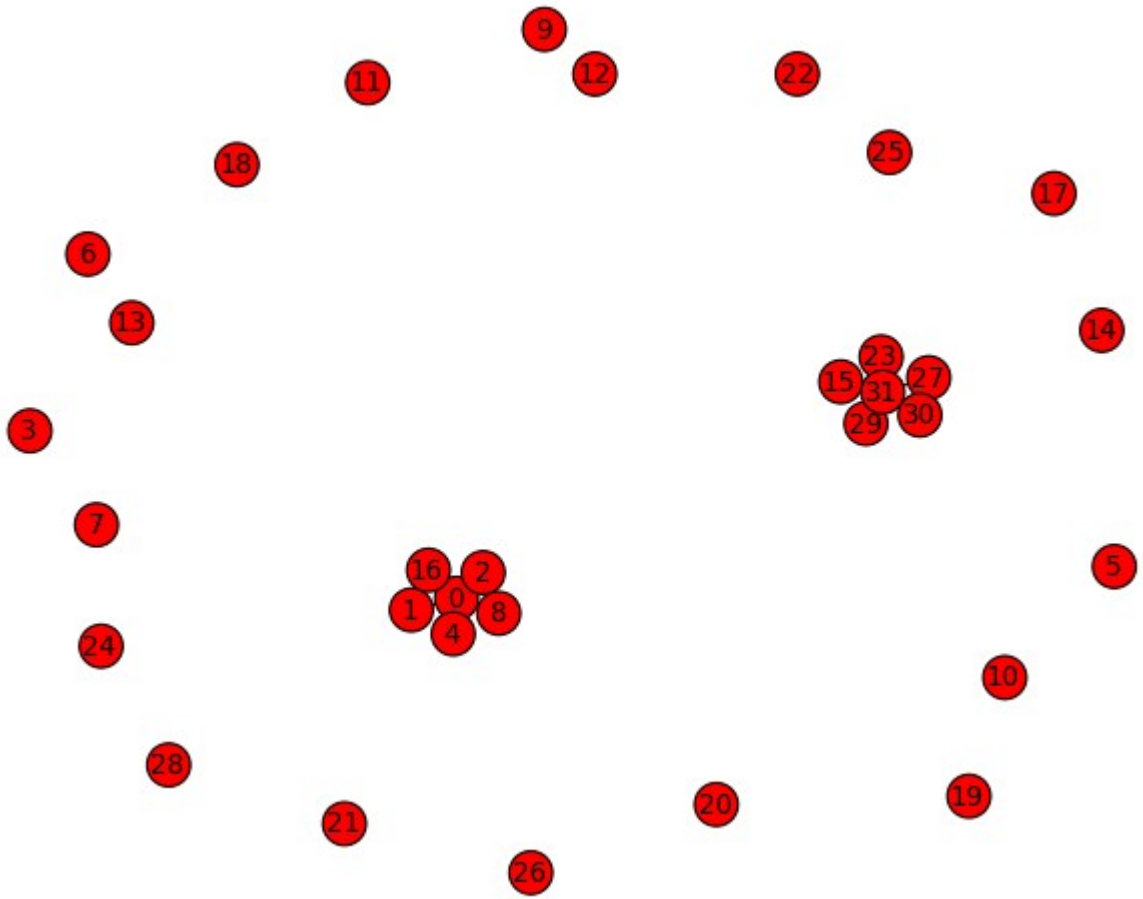

```
p = [ "{0:05b}".format(i) for i in range(2**5)]
pp=map(lambda x: map(eval,list(x)),p)
```

What errors can I recover from?



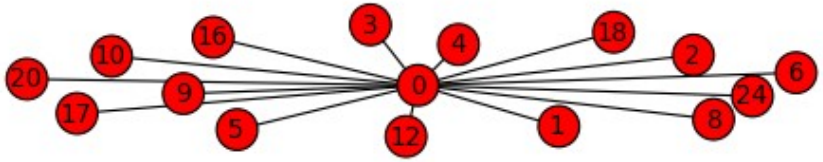

```
def ham(x,y): return sum(map(operator.ne, x,y))
```

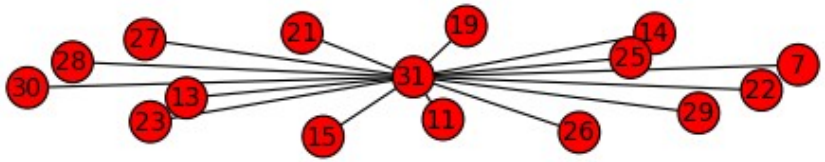
Within 1 hamming distance unit




```
for i in p if ham(i,p[-1])<=1]
for i in p if ham(i,p[0])<=1]
```

Within 2 hamming distance unit






```
 for i in p if ham(i,p[-1])<=2]
 for i in p if ham(i,p[0])<=2]
```

Binary Repetition Codes

11111

00000

Transmit 0 or 1

$$
\begin{aligned}
i_1 + p_1 \qquad\qquad\qquad\qquad &= 0 \\
i_1 \qquad + p_2 \qquad\qquad\qquad &= 0 \\
i_1 \qquad\qquad + p_3 \qquad\qquad &= 0 \\
\vdots \qquad\qquad\qquad\qquad &\vdots \\
i_1 \qquad\qquad\qquad\qquad + p_{n-1} &= 0
\end{aligned}
$$

```
def chk(x):
    return [(x[0]+x[1])%2, (x[0]+x[2])%2, (x[0]+x[3])%2, (x[0]+x[4])
  %2]
```

Binary Repetition Codes (5,1)

```
In [480]: [(i,chk(i),sum(i)) for i in pp if sum(i)<=2]

Out[480]:

[([0, 0, 0, 0, 0], [0, 0, 0, 0], 0),
 ([0, 0, 0, 0, 1], [0, 0, 0, 1], 1),
 ([0, 0, 0, 1, 0], [0, 0, 1, 0], 1),
 ([0, 0, 0, 1, 1], [0, 0, 1, 1], 2),
 ([0, 0, 1, 0, 0], [0, 1, 0, 0], 1),
 ([0, 0, 1, 0, 1], [0, 1, 0, 1], 2),
 ([0, 0, 1, 1, 0], [0, 1, 1, 0], 2),
 ([0, 1, 0, 0, 0], [1, 0, 0, 0], 1),
 ([0, 1, 0, 0, 1], [1, 0, 0, 1], 2),
 ([0, 1, 0, 1, 0], [1, 0, 1, 0], 2),
 ([0, 1, 1, 0, 0], [1, 1, 0, 0], 2),
 ([1, 0, 0, 0, 0], [1, 1, 1, 1], 1),
 ([1, 0, 0, 0, 1], [1, 1, 1, 0], 2),
 ([1, 0, 0, 1, 0], [1, 1, 0, 1], 2),
 ([1, 0, 1, 0, 0], [1, 0, 1, 1], 2),
 ([1, 1, 0, 0, 0], [0, 1, 1, 1], 2)]
```

Make code corrections

```
k=dict([(tuple(chk(i)),i) for i in pp if sum(i)<=2] )
def fix(x):
     return [(i+j)%2 for i,j in zip(k[tuple(chk(x))],x)]
```

```
In [480]: [(i,chk(i),sum(i)) for i in pp if sum(i)<=2]
Out[480]:
[([0, 0, 0, 0, 0], [0, 0, 0, 0], 0),
 ([0, 0, 0, 0, 1], [0, 0, 0, 1], 1),
 ([0, 0, 0, 1, 0], [0, 0, 1, 0], 1),
 ([0, 0, 0, 1, 1], [0, 0, 1, 1], 2),
 ([0, 0, 1, 0, 0], [0, 1, 0, 0], 1),
 ([0, 0, 1, 0, 1], [0, 1, 0, 1], 2),
 ([0, 0, 1, 1, 0], [0, 1, 1, 0], 2),
 ([0, 1, 0, 0, 0], [1, 0, 0, 0], 1),
 ([0, 1, 0, 0, 1], [1, 0, 0, 1], 2),
 ([0, 1, 0, 1, 0], [1, 0, 1, 0], 2),
 ([0, 1, 1, 0, 0], [1, 1, 0, 0], 2),
 ([1, 0, 0, 0, 0], [1, 1, 1, 1], 1),
```

Applying corrections

```
In [48]: for i in pp:
    ....:     (i,fix(i))
    ....:
Out[48]: ([0, 0, 0, 0, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 0, 0, 1], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 0, 1, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 0, 1, 1], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 1, 0, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 1, 0, 1], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 1, 1, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 0, 1, 1, 1], [1, 1, 1, 1, 1])
Out[48]: ([0, 1, 0, 0, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 1, 0, 0, 1], [0, 0, 0, 0, 0])
Out[48]: ([0, 1, 0, 1, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 1, 0, 1, 1], [1, 1, 1, 1, 1])
Out[48]: ([0, 1, 1, 0, 0], [0, 0, 0, 0, 0])
Out[48]: ([0, 1, 1, 0, 1], [1, 1, 1, 1, 1])
```

(7,4) Binary Hamming Code

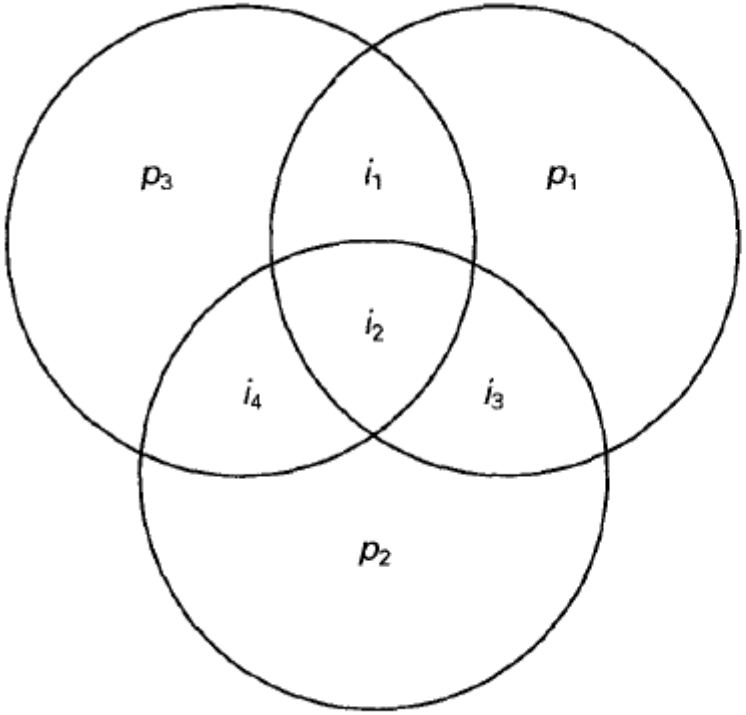
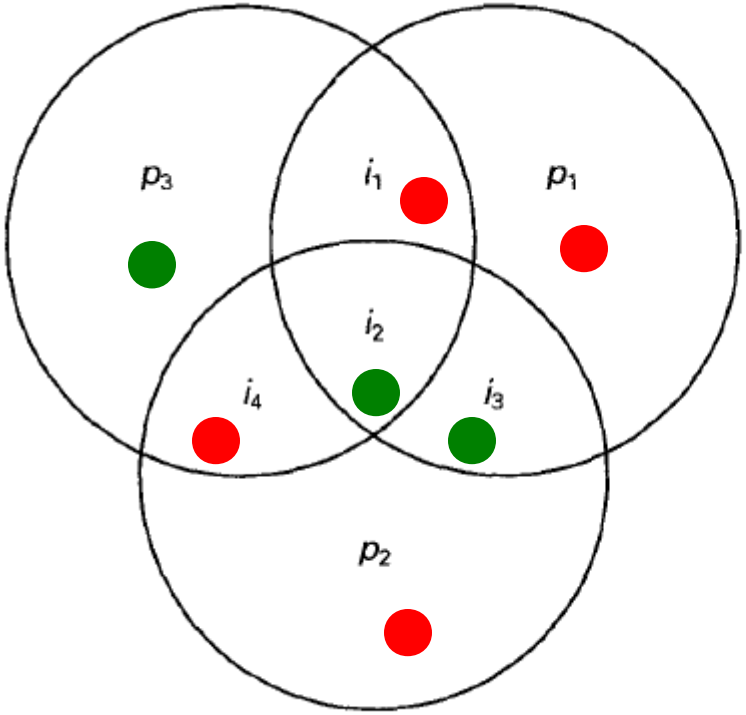


Figure 5.0.2 Venn diagram illustration of (7, 4) binary Hamming code.

(7,4) Binary Hamming Code



$$p_1 + i_1 + i_2 + i_3 = 0,$$

$$p_2 + i_2 + i_3 + i_4 = 0,$$

$$p_3 + i_1 + i_2 + i_4 = 0,$$

$i_1 i_2 i_3 i_4 p_1 p_2 p_3$

(7,4) Binary Hamming Code

```
In [361]: p = [ "{0:07b}".format(i) for i in
   range(2**7)]


In [362]: pp=map(lambda x: map(eval,list(x)),p)


In [363]: [(i,chk(i),sum(chk(i))) for i in pp]

Out[363]:

[([0, 0, 0, 0, 0, 0, 0], [0, 0, 0], 0),

 ([0, 0, 0, 0, 0, 0, 1], [0, 0, 1], 1),

 ([0, 0, 0, 0, 0, 1, 0], [0, 1, 0], 1),

 ([0, 0, 0, 0, 0, 1, 1], [0, 1, 1], 2),

 ([0, 0, 0, 0, 1, 0, 0], [1, 0, 0], 1),

 ([0, 0, 0, 0, 1, 0, 1], [1, 0, 1], 2),

 ([0, 0, 0, 0, 1, 1, 0], [1, 1, 0], 2),
```

(7,4) Binary Hamming Code

```
In [372]: [(i,chk(i),sum(chk(i))) for i in pp if
    sum(chk(i))==0]
```

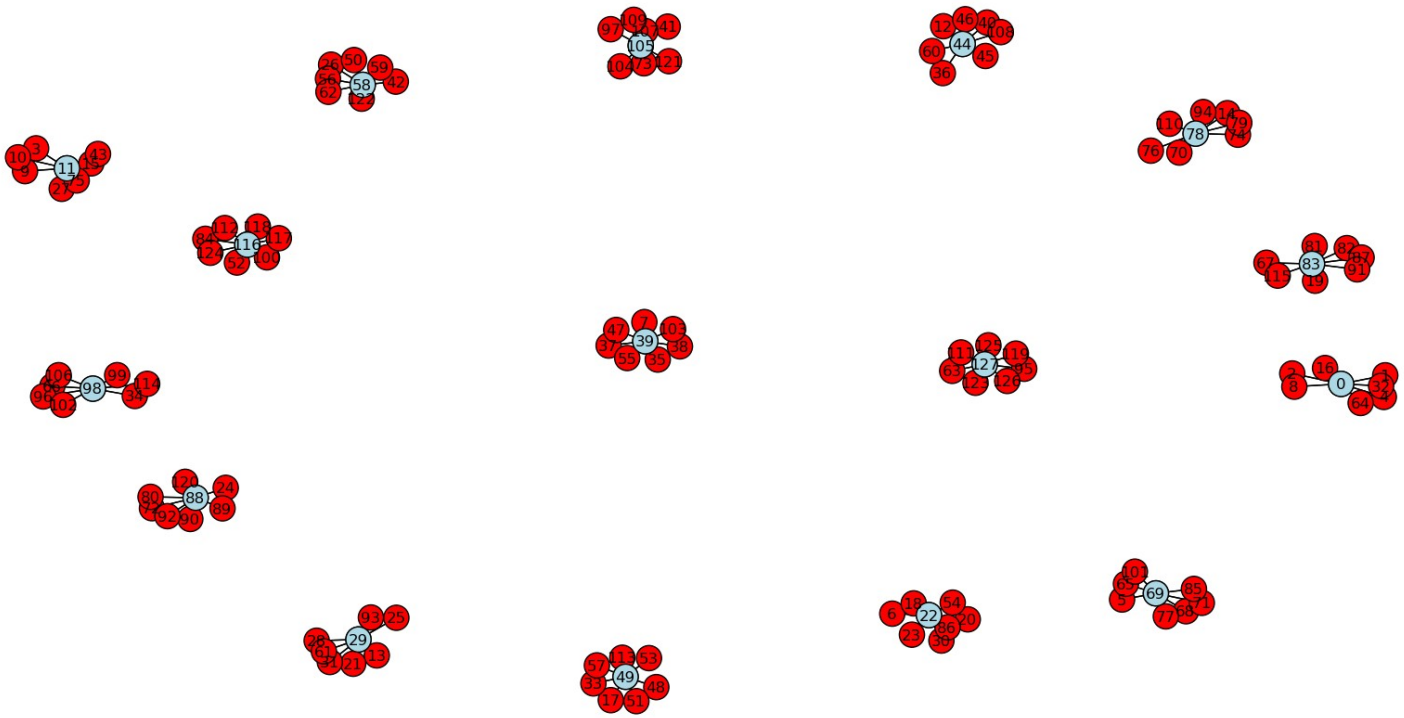
```
Out[372]:
[([0, 0, 0, 0, 0, 0, 0], [0, 0, 0], 0),
 ([0, 0, 0, 1, 0, 1, 1], [0, 0, 0], 0),
 ([0, 0, 1, 0, 1, 1, 0], [0, 0, 0], 0),
 ([0, 0, 1, 1, 1, 0, 1], [0, 0, 0], 0),
 ([0, 1, 0, 0, 1, 1, 1], [0, 0, 0], 0),
 ([0, 1, 0, 1, 1, 0, 0], [0, 0, 0], 0),
 ([0, 1, 1, 0, 0, 0, 1], [0, 0, 0], 0),
 ([0, 1, 1, 1, 0, 1, 0], [0, 0, 0], 0),
 ([1, 0, 0, 0, 1, 0, 1], [0, 0, 0], 0),
 ([1, 0, 0, 1, 1, 1, 0], [0, 0, 0], 0),
 ([1, 0, 1, 0, 0, 1, 1], [0, 0, 0], 0),
 ([1, 0, 1, 1, 0, 0, 0], [0, 0, 0], 0),
 ([1, 1, 0, 0, 0, 1, 0], [0, 0, 0], 0),
 ([1, 1, 0, 1, 0, 0, 1], [0, 0, 0], 0),
 ([1, 1, 1, 0, 1, 0, 0], [0, 0, 0], 0),
 ([1, 1, 1, 1, 1, 1, 1], [0, 0, 0], 0)]
```

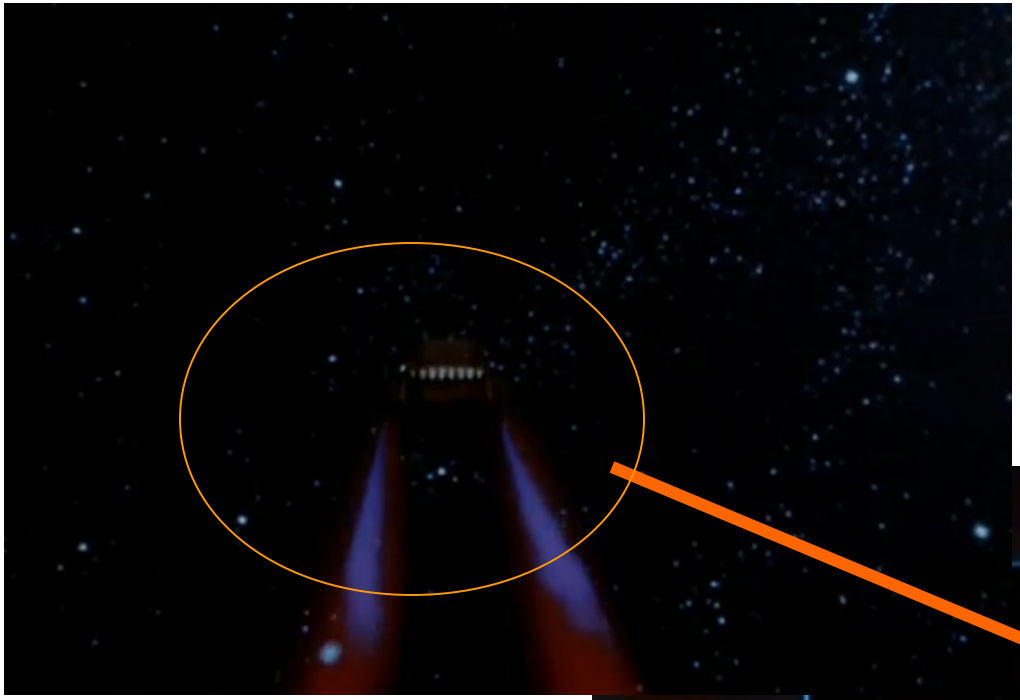
(7,4) Binary Hamming Code

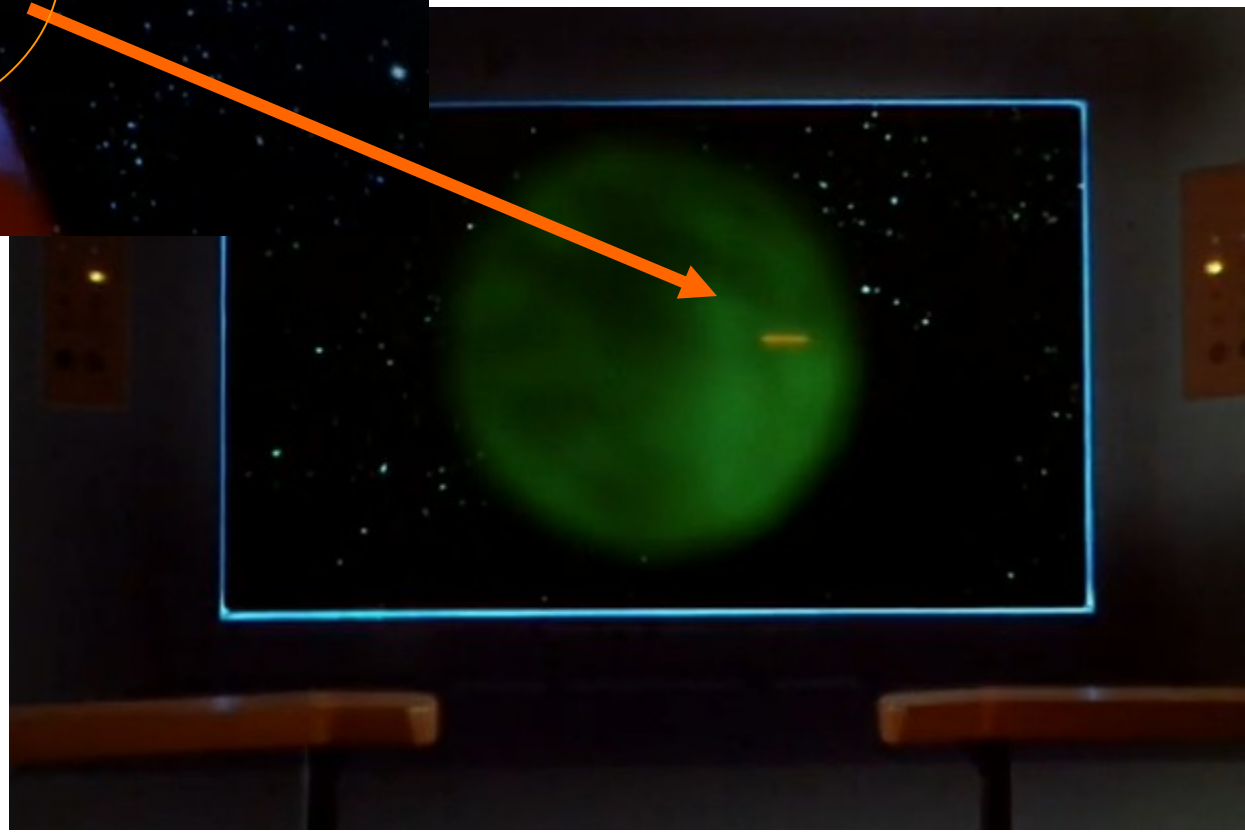
```
In [387]: [(i,chk(i),sum(chk(i))) for i in pp if sum(i)==1]
Out[387]:
[([0, 0, 0, 0, 0, 0, 1], [0, 0, 1], 1),
 ([0, 0, 0, 0, 0, 1, 0], [0, 1, 0], 1),
 ([0, 0, 0, 0, 1, 0, 0], [1, 0, 0], 1),
 ([0, 0, 0, 1, 0, 0, 0], [0, 1, 1], 2),
 ([0, 0, 1, 0, 0, 0, 0], [1, 1, 0], 2),
 ([0, 1, 0, 0, 0, 0, 0], [1, 1, 1], 3),
 ([1, 0, 0, 0, 0, 0, 0], [1, 0, 1], 2)]
```

```
In [402]: [(i,chk(i),sum(i)) for i in pp if sum(i)==2]
Out[402]:
[([0, 0, 0, 0, 0, 1, 1], [0, 1, 1], 2),
 ([0, 0, 0, 0, 1, 0, 1], [1, 0, 1], 2),
 ([0, 0, 0, 0, 1, 1, 0], [1, 1, 0], 2),
 ([0, 0, 0, 1, 0, 0, 1], [0, 1, 0], 2),
 ([0, 0, 0, 1, 0, 1, 0], [0, 0, 1], 2),
 ([0, 0, 0, 1, 1, 0, 0], [1, 1, 1], 2),
 ([0, 0, 1, 0, 0, 0, 1], [1, 1, 1], 2),
 ([0, 0, 1, 0, 0, 1, 0], [1, 0, 0], 2),
 ([0, 0, 1, 0, 1, 0, 0], [0, 1, 0], 2),
 ([0, 0, 1, 1, 0, 0, 0], [1, 0, 1], 2),
 ([0, 1, 0, 0, 0, 0, 1], [1, 1, 0], 2),
 ([0, 1, 0, 0, 0, 1, 0], [1, 0, 1], 2),
 ([0, 1, 0, 0, 1, 0, 0], [0, 1, 1], 2),
 ([0, 1, 0, 1, 0, 0, 0], [1, 0, 0], 2),
 ([0, 1, 1, 0, 0, 0, 0], [0, 0, 1], 2),
 ([1, 0, 0, 0, 0, 0, 1], [1, 0, 0], 2),
 ([1, 0, 0, 0, 0, 1, 0], [1, 1, 1], 2),
 ([1, 0, 0, 0, 1, 0, 0], [0, 0, 1], 2),
 ([1, 0, 0, 1, 0, 0, 0], [1, 1, 0], 2),
 ([1, 0, 1, 0, 0, 0, 0], [0, 1, 1], 2),
 ([1, 1, 0, 0, 0, 0, 0], [0, 1, 0], 2)]
```

(7,4) Binary Hamming Code







Python Iterators

An **iterator** is an object representing a stream of data

```
>>> L = [1,2,3]
>>> it = iter(L)
>>> print it
<iterator object at 0x8116870>
>>> it.next()
1
>>> it.next()
2
>>> it.next()
3
>>> it.next()
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
StopIteration
>>>
```

Python Custom iterators

```
class Count(object):
    def __init__(self):
        self.x = 10
    def __iter__(self):
        return self
    def next(self):
        print 'am in next'
        self.x -= 1
        if self.x < 0:
           raise StopIteration
        else:
           return self.x
```

Python List Comprehesions

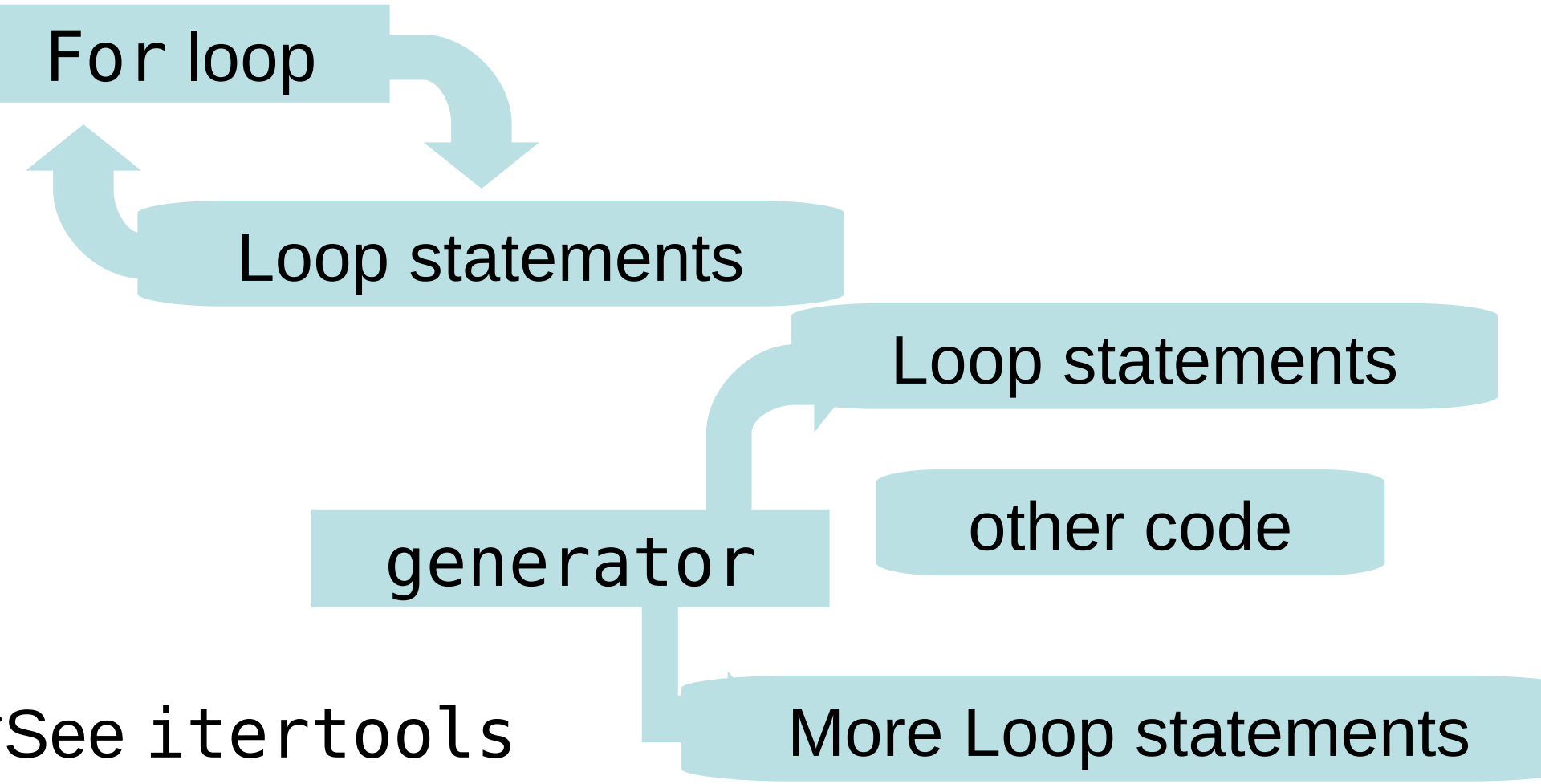
```
>>> seq1 = 'abc'
>>> seq2 = (1,2,3)
>>> [ (x,y) for x in seq1 for y in seq2]
[('a', 1), ('a', 2), ('a', 3),
 ('b', 1), ('b', 2), ('b', 3),
 ('c', 1), ('c', 2), ('c', 3)]
```

Generators

- Produces a stream of on-demand values
- Only executes on `next()`
- `yield()` produces a value, but saves the function's state for later
- Consumable (use once-only)

```
def generate_ints(N):
    for i in range(N):
        yield i
gen=( i for i in range(3) )
```

Generators allow non-linear looping



See `itertools`

Generator: Permutation Example

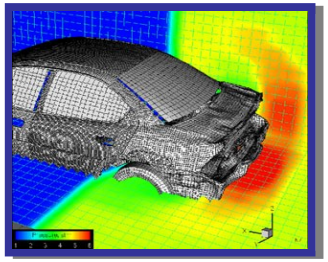
```
>>> n=0
>>> for i in itertools.permutations(range(10)):
    n+=1

>>> scipy.factorial(10)
array(3628800)

>>> scipy.factorial( 10)*4
14515200
```

Each element generated on-demand, not stored in a list (would not fit!)

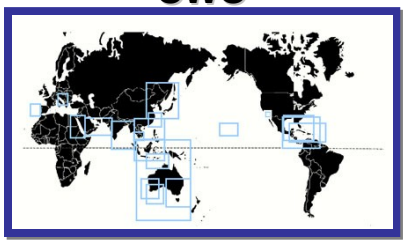
HPCMP Functional Areas (CTAs)

Climate/Weather/Ocean Modeling (CWO)
Computational Chemistry and Materials Science (CCM)
Computational Electromagnetics and Acoustics (CEA)
Computational Electronics and Nanoelectronics (CEN)
Computational Fluid Dynamics (CFD)
Computational Structural Mechanics (CSM)
Environmental Quality Modeling and Simulation (EQM)
Forces Modeling and Simulation (FMS)
Integrated Modeling and Test Environments (IMT)
Signal/Image Processing (SIP)

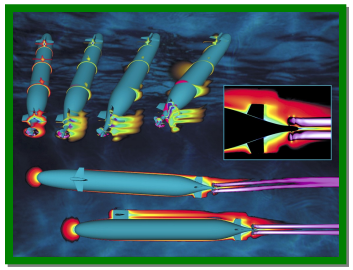
CSM

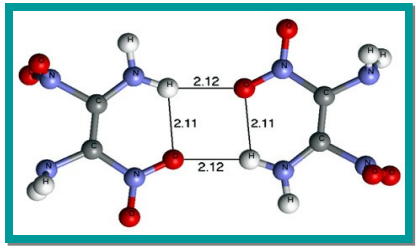


CWO

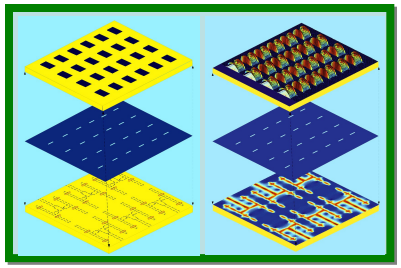


CFD





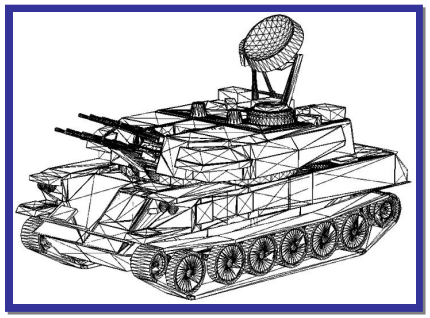
CCM



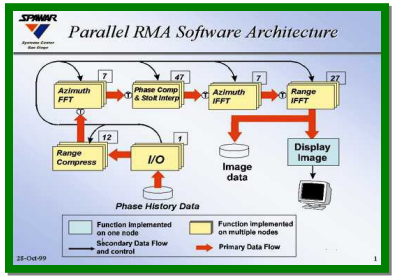
CEN

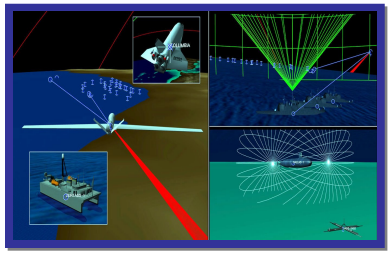
FMS

CEA



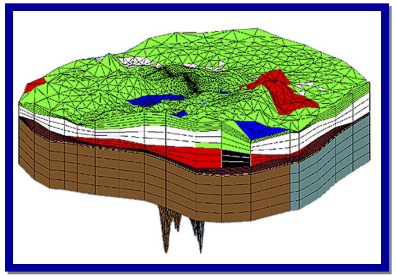
SIP

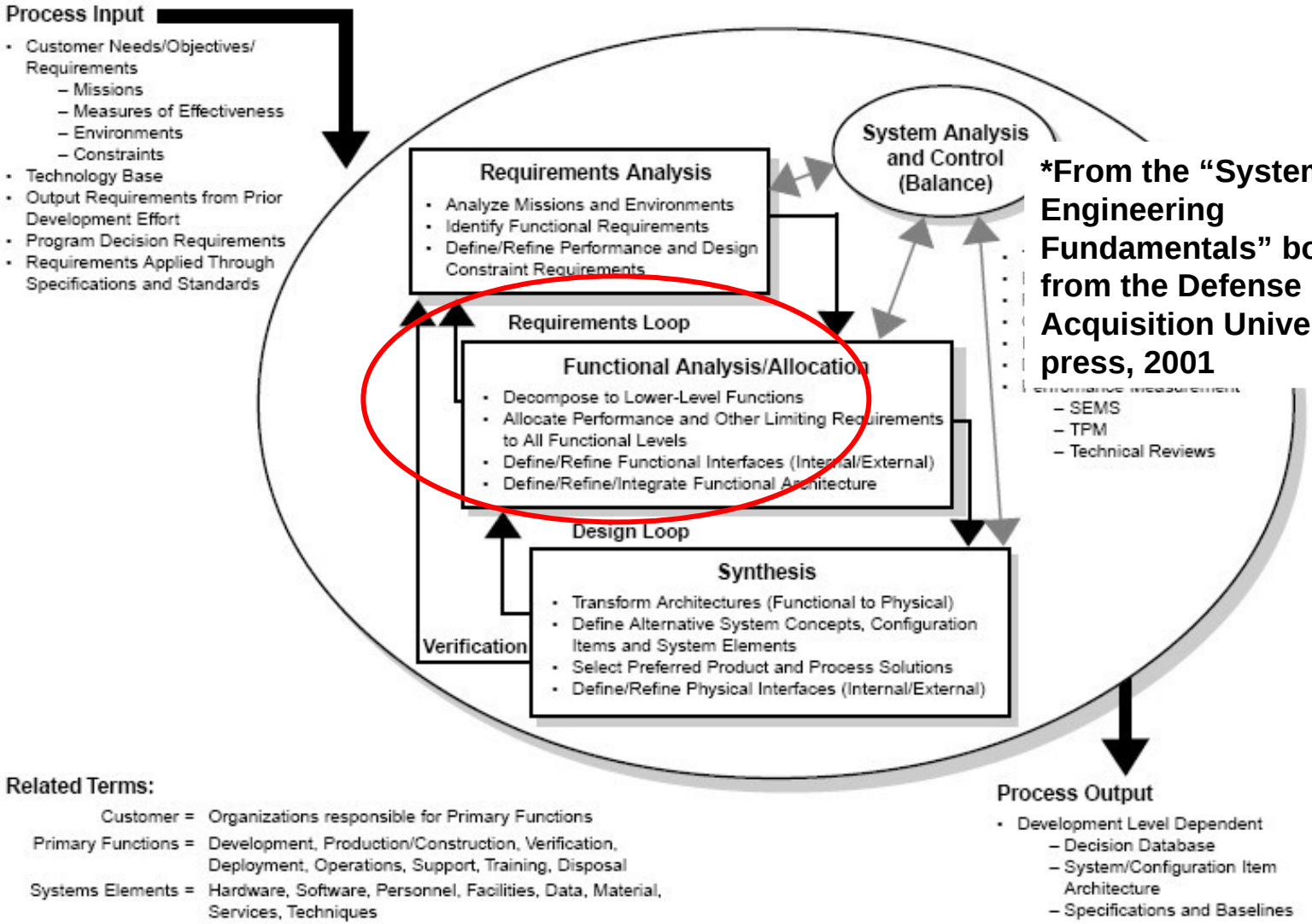




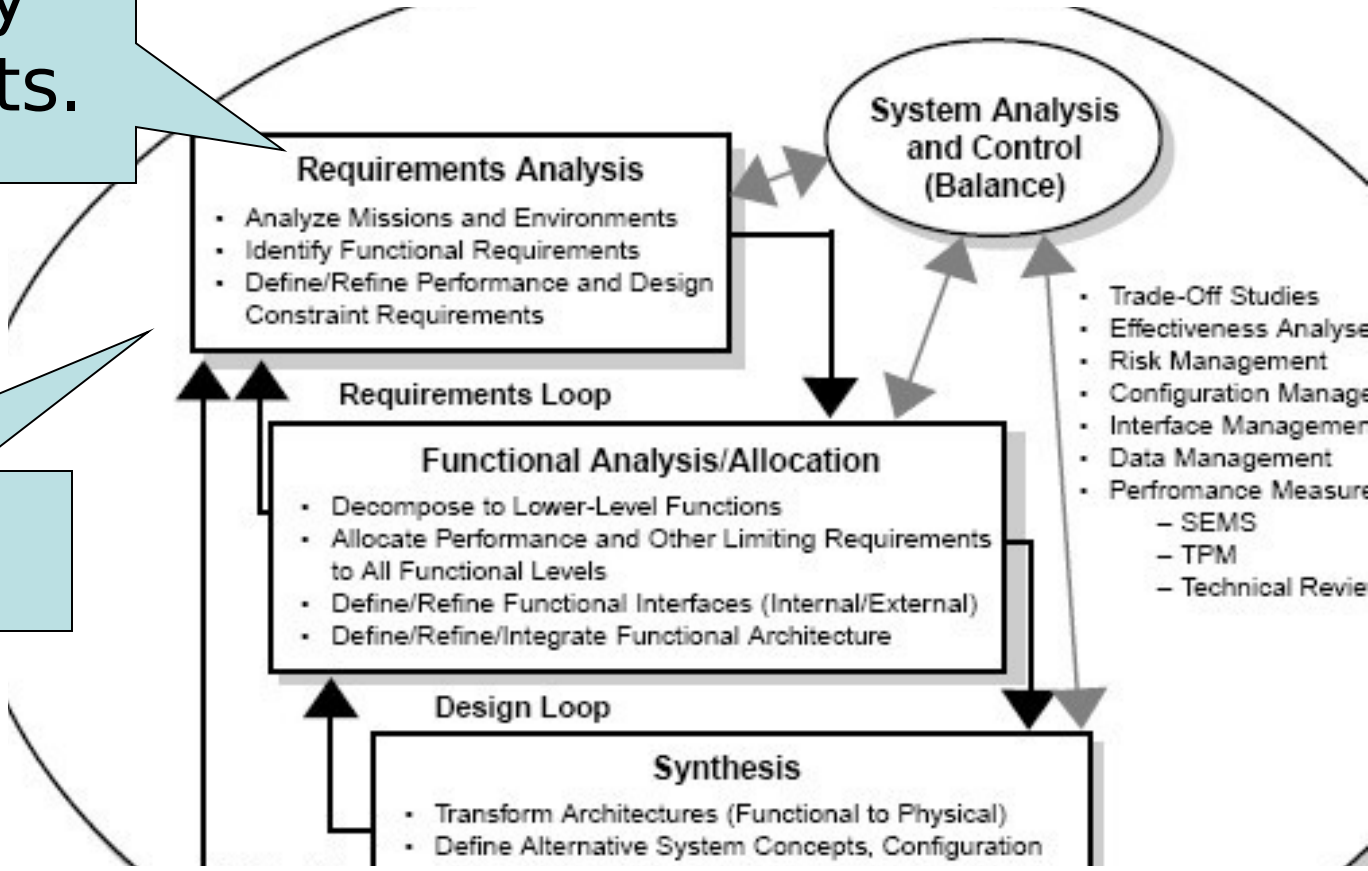
IMT

EQM





***From the "System Engineering Fundamentals" book from the Defense Acquisition University press, 2001**

90% costs defined by requirements.

Minimize Risk!