Applications of Fourier Transforms and Convolutions in Optics: Limited to the Application of the Diffraction of Light

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Final Capstone Paper

April 27, 2007

SM472

Goals: To examine the applications of Fourier transforms and convolutions in Fourier optics with specific attention devoted to the Fourier lens. The fundamental properties of Fourier transformations coupled with convolutions are an integral part of Fourier optics. Several properties of Fourier transforms are reviewed and directly related to actual optical demonstrations in the laboratory.

Acknowledgments: I would like to thank Professor Tankersley for all his guidance and assistance in completion of this analysis. Professor Tankersley's knowledge of optics and mathematics was an enormous asset. Without his patience, tireless dedication, and expertise, this paper would not have been possible.

Nevertheless, before delving into the complex realm of Fourier optics, a certain amount of mathematical introduction and explanation is required. The theorems, definitions, and proofs are presented to introduce the fundamentals necessary for expansion from theoretical to practical. But who was Fourier?

Joseph Fourier had been quite gifted in mathematics early in his teenage years. However, he had decided to embark upon a career in the priesthood in France. He was very conflicted between mathematics and religion and would ultimately deny his religious vows. Fourier became entangled in the French revolution and found himself placed in prison awaiting the guillotine. But, he was eventually freed. Fourier then began teaching at the Collège de France and worked side by side with Lagrange and Laplace. After teaching, Fourier joined Napoleon's Army and was in charge of new innovations in Egypt. While in Egypt, Fourier derived the heat equation. After his military service, which was closely dictated by Napoleon until his defeat, he returned to France and continued his research. Fourier's life was not without controversy. In fact, Jean-Baptiste Biot had attempted to claim the discovery of Fourier's heat theory and Poisson disputed his methods. [1]

http://www-history.mcs.st-andrews.ac.uk/history/Posters2/Fourier.html

What is Fourier analysis? Fourier analysis allows a signal or function to be separated or decomposed into components made up of simpler inputs for linear systems. The individual inputs have a response associated with them and that is the sum of the responses to the individual points.[2] The process described above is referred to as convolution or superposition in a linear system.

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[1] MacTutor History of Mathematics, Jan. 1997, University of St. Andrews, 24 Jan. 2007 < http://www-history.mcs.st-andrews.ac.uk/Biographies/Fourier.html>.

[2] Joseph W. Goodman, Introduction to Fourier Optics (New York: McGraw-Hill, 1968) 7.

Still confused as to what convolution really is?

Steward explains convolution as, "the distribution of one function in accordance with the law specified by another function."[3] We might say the smearing of one function with another. Essentially, each ordinate of a function is multiplied by another function and summed.[4] Nevertheless, the functions are assumed linear and invariant or "stationary" because transformations do not change the function, thus preserving length.[5] In brief, convolutions are overlaps that are the result of spreading or smearing of one function using the rule or operation of another.

Definition of convolution and theorem[6]

If f and g are two functions, with $\|f\|_1$ and $\|g\|_1$ finite, where

$\|f\|_1$ is defined as $\int_{-\infty}^{\infty} |f(x)|\,dx$ and $\|g\|_1$ is defined as $\int_{-\infty}^{\infty} |g(x)|\,dx$

the **convolution** of f and g is denoted by $f * g$ and is defined by:

$$f * g(x) = \int_{-\infty}^{\infty} f(s)\, g(x-s)\, ds$$

Convolutions are tremendously powerful because of the properties they possess, such as:[7]

1) Commutivity- $f * g = g * f$

2) Associativity- $f * (g * h) = (f * g) * h$

3) Distribution of Addition- $f * (g + h) = (f * g) + (f * h)$

What is a Fourier transform? A Fourier transform represents a function as a sum of characteristic frequencies or pure frequencies. But, the caveat is that Fourier transforms are used with Lebesgue square integrable functions. Physicists normally refer to the Fourier transform as the Fourier spectrum and tend to look at a two-dimensional version where light is illuminated on a plane screen perpendicular to the direction of propagation.[8] A standard convention is to run the coordinate z in the direction of the propagation and to use x and y in the plane of the screen.

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[3] E.G. Steward, Fourier Optics: an Introduction, 2nd Ed. (New York: Halstead, 1987) 85.

[4] Steward 85-86.

[5] Steward 102.

[6] Walker 149, 173.

[7] Steward 85.

[8] Goodman 5, 86.

Definition of Fourier Transform [9]

Let $\|\mathbf{f}\|_2$ be defined as $\int_{-\infty}^{\infty}|f(x)|\,dx$ and $\|\mathbf{f}\|_2$ as$[\int_{-\infty}^{\infty}|f(x)|^2\,dx\,]^{\frac{1}{2}}$

Given a function for which $\|\mathbf{f}\|_2$ is finite, the **Fourier transform** of f is denoted by $\hat{f}$ and is defined as a function of **u** by

$$\hat{f}(u) = \int_{-\infty}^{\infty} f(x)\; e^{-i2\pi ux}dx$$

Gaussian functions ($\sqrt{\frac{a}{\pi}}e^{-ax^2}$) are of particular interests because the Fourier transform of a Gaussian function is another Gaussian function. The Gaussian and its Fourier transform have the same relative shape and is a good example for studying the relative widths of a function and its transform. This result is of great interest to physicists as a 'good' or 'nice' laser beam has a transverse profile that is Gaussian. The Gaussian profile diffracts into a Gaussian profile and essentially projects itself, thus maintaining its relative transverse profile. Note the derivation below to see for yourselves.

Example[10]

let $f(x) = \sqrt{\frac{a}{\pi}}e^{-ax^2}$, $\Im\{f(x)\} = ?$

$F(k) = \int_{-\infty}^{\infty}(\sqrt{\frac{a}{\pi}}e^{-ax^2})e^{ikx}dx$, now complete the square:

$-ax^2 + ikx$ becomes $-(x\sqrt{a} - ikx)^2 - \frac{k^2}{4}$

Now let $(x\sqrt{a} - ikx) = \beta$,

Next, square the integral so we can place it into polar form:

$\frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}}e^{-\frac{k^2}{4a}}(\int_{-\infty}^{\infty}e^{-\beta^2}d\beta)\,(\int_{-\infty}^{\infty}e^{-\beta^2}d\beta)$ let the second $\beta = \alpha$

$F(k) = \int_{-\infty}^{\infty}\sqrt{\frac{a}{\pi}}e^{-\beta^2-\frac{k^2}{4a}}d\beta \;\Rightarrow F(k) = \frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}}e^{-\frac{k^2}{4a}}\int_{-\infty}^{\infty}e^{-\beta^2}d\beta$

$= \frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}}e^{-\frac{k^2}{4a}}(\int_{-\infty}^{\infty}e^{-\beta^2}d\beta)\,(\int_{-\infty}^{\infty}e^{-\alpha^2}d\alpha)$

$= \frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}}e^{-\frac{k^2}{4a}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-\beta^2-\alpha^2}d\beta d\alpha$

$= \beta^2 + \alpha^2 = r^2,\; d\beta d\alpha = rdrd\theta$

$= \frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}}e^{-\frac{k^2}{4a}}\int_0^{2\pi}\int_0^{\infty}e^{-r^2}drd\theta$

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[9] Walker 149-150.

[10] Eugene Hecht, Optics, 2nd Ed. (Reading MA, Addison-Wesley) 474.

$= \frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}} e^{-\frac{k^2}{4a}} (\int_0^{2\pi} d\theta)(\int_0^{\infty} e^{-r^2} dr)$

let $u = r^2$, $du = 2rdr$, so $rdr = \frac{1}{2} du = dr$

$(\int_0^{2\pi} d\theta)(\frac{1}{2}\int_0^{\infty} e^{-u^2} du) = \pi,$

but remember we squared the integral $\Rightarrow \sqrt{\pi}$

$= (\frac{\sqrt{\frac{a}{\pi}}}{\sqrt{a}} e^{-\frac{k^2}{4a}})(\sqrt{\pi})$

$\Rightarrow \frac{\sqrt{\pi}\sqrt{\frac{a}{\pi}}}{\sqrt{a}} e^{-\frac{k^2}{4a}} = e^{-\frac{k^2}{4a}}$

Note : the integration of $\sqrt{\frac{a}{\pi}} e^{-\beta^2}$ is possible along a contour parallel to the real axis because it can be shifted to a contour along the real axis.

With the Fourier transform defined, the inverse seems to follow naturally. The inverse Fourier transform is actually the Fourier transform with positive exponent.

Definition of Inverse Fourier Transform[11]

If f is continous and $\|f\|_1$ and $\|\hat{f}\|_1$ are both finite, then

$f(x) = \int_{-\infty}^{\infty} \hat{f}(u)\, e^{i2\pi ux} dx$ for all $x \in \mathbb{R}$

As previously mentioned, convolution and Fourier transforms are interrelated in our context. However, with a slight modification to the definition of convolution, the relation is easily and noticeable and applicable. Be aware, that convolution is also referred to as "the folding product" and "composition product" by physicists.[12]

Fourier transforms have four very important properties which allow for simplification and useful manipulation in both physics and mathematics. These properties are linearity, scaling, shifting, and modulation, which are defined on the following page.

[11] Walker 160.

[12] Steward 82.

Linearity Theorem[13] – For ∀a,b where a and b are constants, the Fourier transform satisfies $af + bg \xrightarrow{\Im} a\hat{f} + b\hat{g}$

Scaling Theorem – For each positive constant p,

$f(\frac{x}{p}) \xrightarrow{\Im} p\hat{f}(pu)$ and $f(px) \xrightarrow{\Im} \frac{1}{p}\hat{f}(\frac{u}{p})$

Modulation Theorem – For each real constant c,

$f(x)e^{-i2\pi cu} \xrightarrow{\Im} \hat{f}(u-c)$

Shift Theorem – For each real constant c,

$f(x-c) \xrightarrow{\Im} \hat{f}(u)e^{-i2\pi cu}$

PROOF [14] (Scaling Theorem)

let $s = \frac{x}{p}$ so $f(\frac{x}{p}) \xrightarrow{\Im} \int_{-\infty}^{\infty} f(\frac{x}{p})\, e^{-i2\pi ux} dx = \int_{-\infty}^{\infty} f(s)\, e^{-i2\pi u(ps)} d(ps)$

$= p\int_{-\infty}^{\infty} f(s)\, e^{-i2\pi(pu)s} ds = p\hat{f}(pu)$

The second part of the proof follows with minor manipulation.

PROOF[15] (Modulation Theorem)

note $e^{i2\pi cx}\ e^{-i2\pi ux} = e^{-i2\pi(u-c)x}$ so

$f(x)e^{i2\pi cx} \xrightarrow{\Im} \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi(u-c)x} dx = \hat{f}(u-c)$

PROOF[16] (Shift Theorem)

let $s = x - c$ so $f(x-c) \xrightarrow{\Im} \int_{-\infty}^{\infty} f(x-c)\, e^{-i2\pi ux} dx = \int_{-\infty}^{\infty} f(s)\, e^{-i2\pi u(s+c)} ds$

$= \int_{-\infty}^{\infty} f(s)\, e^{-i2\pi us} ds\ e^{-i2\pi cu} = \hat{f}(u)\, e^{-i2\pi cu}$

The modulation theorem is very advantageous in optics because it shows that multiplication by a linear phase creates a translation and the shift theorem shows that the inverse is true. When a translation is multiplied by a linear phase as in the case of the Gaussian, the result is another Gaussian.[17]

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[13] Walker 155.

[14] Walker 156.

[15] Walker 156.

[16] Walker 156.

[17] Hecht 474.

Example of a Gaussian [Shift theorem]:

Start with:

$$\int_{-\infty}^{\infty} e^{i\lambda x} f(x+a)\, dx \xrightarrow{\Im} e^{ika}\, \hat{f}(k-\lambda)$$

Thus, multiplying by a linear phase creates a translation

Then,

$$\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\Delta}} e^{ikx} dx \xrightarrow{\Im} e^{\frac{-k^2\Delta^2}{2}}$$

Now we can use the above to conclude

$$\int_{-\infty}^{\infty} e^{\frac{-(x^2-a)^2}{2\Delta^2}} e^{ikx} dx \xrightarrow{\Im} e^{ika}\, \hat{f}(k-k_0) = e^{\frac{-(k-k_0)^2\Delta^2}{2}}$$

Seemingly difficult transformations become manageable.

We begin with an introduction of convolution over a finite interval using the Fourier series, which is later expanded to an infinite interval using the Fourier transform.

The function f ∗ P is the **convolution over** [-L,L] **of f and P**, (where P is $\sum_{n=-\infty}^{\infty} F_n e^{i\pi nx/L}$), defined by:[18] $f * P(x) = \frac{1}{2L}\int_{-L}^{L} f(s)\, P(x-s)\, ds$

PROOF[19]

Given $f * P$ exists, compute the nth Fourier coefficent, p_n

$$p_n = \tfrac{1}{2L}\int_{-L}^{L} f * P(x)\, e^{-i\pi nx/L}\, dx$$

$$= \tfrac{1}{2L}\int_{-L}^{L} [\tfrac{1}{2L}\int_{-L}^{L} f(s)\, P(x-s)\, ds]\, e^{-i\pi nx/L} dx$$

However, $e^{-i\pi nx/L}$ is not effected by $\int_{-L}^{L} f(s)\, P(x-s)\, ds$, and we can reverse the order of integration because the integrals are assumed to be convergent, and the integrations are over finite intervals. The limits are constants, that is, no limit depends on the other integration variable so the order may be interchanged. Thus becoming:

$$p_n = \tfrac{1}{2L}\int_{-L}^{L} f(s)[\tfrac{1}{2L}\int_{-L}^{L} P(x-s)\, e^{-i\pi nx/L} dx]\, ds$$

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[18] James Walker, Fast Fourier Transforms, 2nd Ed. (Boca Raton: CRC, 1987) 117.

[19] Walker 117-118.

Now substitute (x-s) + s into $e^{-i\pi nx/L}$ and factor out $e^{-i\pi ns/L}$

$$p_n = \tfrac{1}{2L}\int_{-L}^{L} f(s)\, e^{-i\pi ns/L}[\tfrac{1}{2L}\int_{-L}^{L} P(x-s)\, e^{-i\pi n(x-s)/L}dx]\, ds$$

Now, complete a change of variables and let (x-s) = υ:

$$p_n = \tfrac{1}{2L}\int_{-L}^{L} f(s)\, e^{-i\pi ns/L}[\tfrac{1}{2L}\int_{-L-s}^{L-s} P(\upsilon)\, e^{-i\pi n\upsilon/L}d\upsilon]\, ds$$

This becomes:

$$p_n = \tfrac{1}{2L}\int_{-L}^{L} f(s)\, e^{-i\pi ns/L}[\tfrac{1}{2L}\int_{-L}^{L} P(\upsilon)\, e^{-i\pi n\upsilon/L}d\upsilon]\, ds$$

Which is true because P(υ) $e^{-i\pi n\upsilon/L}$ has a period of 2L so $\int_{-L-s}^{L-s}$ becomes $\int_{-L}^{L}$

However, we defined $\tfrac{1}{2L}\int_{-L}^{L} P(\upsilon)\, e^{-i\pi n\upsilon/L}d\upsilon$ above as F_n :

$F_n = \tfrac{1}{2L}\int_{-L}^{L} P(x)\, e^{-i\pi nx/L}dx$ and $p_n = \tfrac{1}{2L}\int_{-L}^{L} f(s)\, e^{-i\pi ns/L}F_n\ ds$

But, recall $c_n = \tfrac{1}{2L}\int_{-L}^{L} f(x)\, e^{-i\pi nx/L}dx$

Therefore, $p_n = c_n F_n$

Convolution Theorem[20]: The Fourier Transform of the convolution of two functions f and g is the multiplication of the Fourier transform of f with the Fourier transform of g.

$f * g$ is $(\hat{f})(\hat{g})$

PROOF[21] (The Convolution Theorem and its Applications)

Note : The symbol ($\Im$) is used to denote a Fourier transform ($\hat{f}$)

$$\Im(f(x) * g(x)) = \Im(\int_{-\infty}^{\infty} f(x)g(u-x)\, dx)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(u-x)\, dx\, e^{-2\pi isu}du$$

Let $w = u - x$

$$\Im(f(x) * g(x)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(w)\, e^{-2\pi is(x+w)}dx\, dw$$

$$= \Im(f)\Im(g) = \hat{f}(u)\, \hat{g}(u)$$

[20] Walker 178.

[21] Hecht 491.

The convolution theorem is a very powerful statement because of the relationship it forms between Fourier transforms and convolutions. The theorem states that the Fourier transform of a convolution is the product of the Fourier transforms of each of the individual functions. The convolution theorem allows a convolution of two functions to be taken by multiplying the Fourier transform of the respective functions, thus simplifying an immense amount of calculation. However, the result must have a Fourier inverse.

With a slight modification, the statement relates the inverse Fourier transform of two functions to the convolution of two functions. The theorem is just as powerful as the previous version because it reveals that whenever an inverse Fourier transform is done on two functions, there is a corresponding convolution of two functions. The theorem unlocks a very important and complex idea that's within our context.

Convolution Theorem[22] **(Inverse) :** The inverse Fourier transform of $\hat{f}\,\hat{g}$ is the convolution of f*g

Such a remarkable theorem requires a proof because of its possibilities and applications in mathematics, most specifically optics.

PROOF[23]

Using the inverse Fourier Transform:

$$\Im^{-1}(\hat{f}\,\hat{g})(x) = \int_{-\infty}^{\infty} \hat{f}(u)\hat{g}(u)\, e^{2i\pi ux}\, du$$

$$= \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} f(s)\, e^{-2i\pi us} ds]\, \hat{g}(u)\, e^{2i\pi ux} du$$

Now interchange the integrals and add the exponents:

$$\Im^{-1}(\hat{f}\,\hat{g})(x) = \int_{-\infty}^{\infty} f(s)[\int_{-\infty}^{\infty} \hat{g}(u)\, e^{2i\pi u(x-s)} du]\, ds$$

This is the integral of $\hat{g}(u)$, with the variable (x-s) so we get g(x-s) for that integral.

Thus, $\Im^{-1}(\hat{f}\,\hat{g})(x) = f * g$

[22] Walker 178.

[23] Walker 177

Parseval's equality[24] **:**

In physics, the relation is normally written as:

Given: $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(x)\ e^{ikx} dx$ and $\hat{f}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\ e^{-ikx} dx$

$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{g}(x)\ e^{imx} dx$ and $\hat{g}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x)\ e^{-imx} dx$

Then **Parseval's *e*quality** is:

$\int_{-\infty}^{\infty} \overline{g(x)} f(x)\ dx = \int_{-\infty}^{\infty} \overline{\hat{g}(k)}\ \hat{f}(k)\ dk$ (where $\overline{g(x)}$ and $\overline{f(x)}$ is the complex conjugate)

On a basic level, the equality relates the square of the amplitude of the incoming light to the square of the light amplitude in the diffraction plane. However, Parseval's equality is essentially a statement of energy conservation because the power leaving the object plane is equal to the power incident on the transform plane.[25] Yet, the integrand allows calculation of the inner product in terms of its respective components.

<u>Applications, Illustrations and Examples:</u>

As shown, convolutions are overlaps that are the result of spreading or smearing of one function by another. Convolution facilitates representing a regular complex pattern to as a basic pattern repeated in accordance with an array function. Nevertheless, all the power does not rest in convolution alone, but jointly between convolution and Fourier transforms because in our context, they are tightly interlaced.

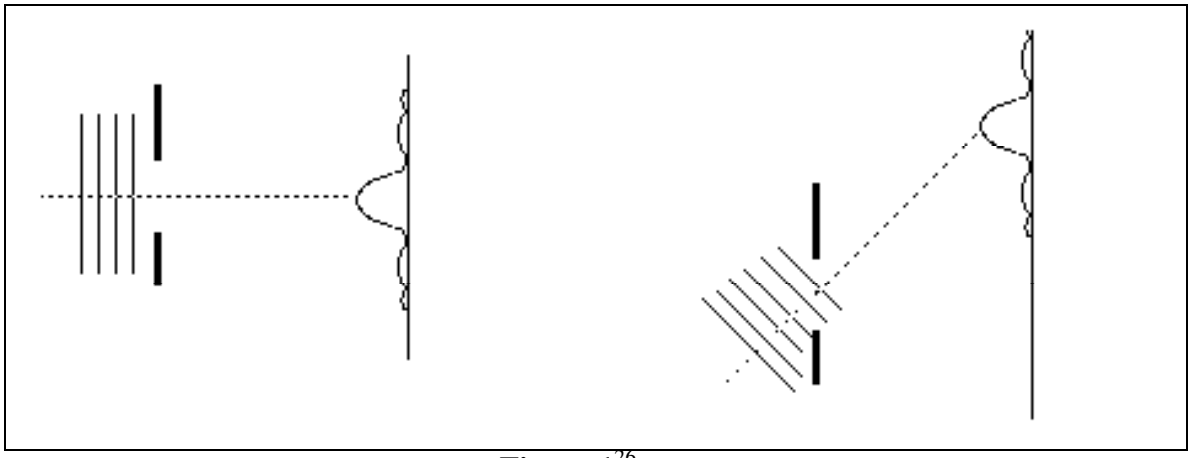

Figure 1[26]

[24] Prof. Tankersley

[25] Hecht 499.

[26] Diagram obtained from Prof. Tankersley

Figure 1 shows a plane wave entering through an aperture. The diffraction pattern of the plane wave through the slit is the square of Fourier transform, which explains why the pattern has only positive components. If the light is shifted at the aperture, the result is a modulation of the square of the Fourier transform. Nevertheless, the entering light maintains the same amplitude (square of the amplitude), visibly displaying Parseval's equality and properties of Fourier transforms.

Fourier transformations themselves are a necessary tool for physicists and mathematicians because of their properties, but coupling with "convolution" allows for remarkable results in physics and mathematics, as seen below. The diagram depicts the top-hat function convolved with another top-hat function creating the triangle function which is precisely how a spectrometer works.

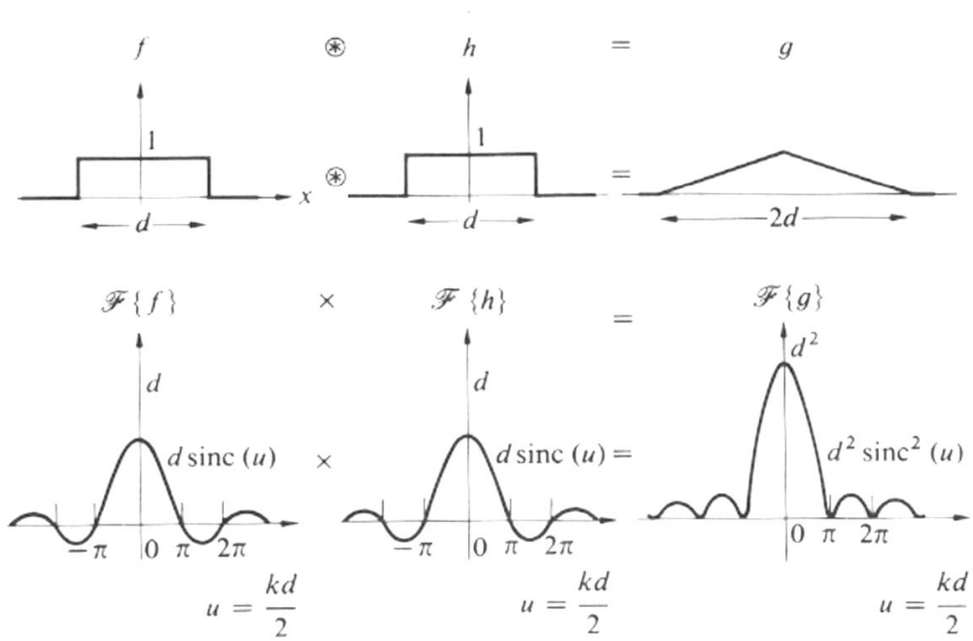

Figure 2[27]

The spectrometer in Figure 3 uses a rotating grating to convolve the light with itself. The light, represented by the top-hat function, convolves with itself and creates the triangle function. The light exits the slit and the intensity pattern varies with time because the apparatus is rotating. The resultant pattern is the top hat function.

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[27] Hecht 492.

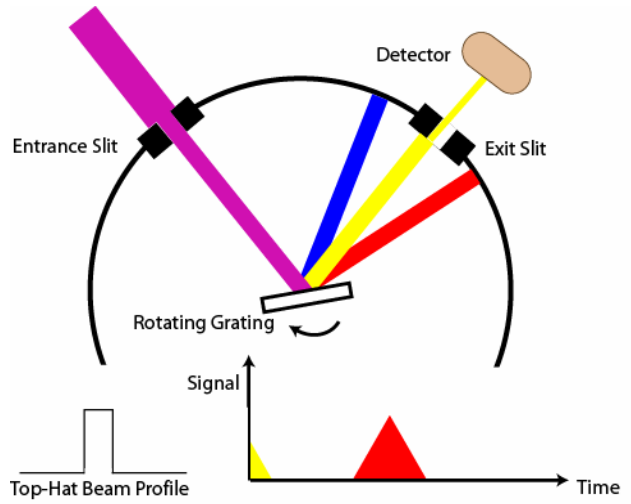


Figure 3[28]

The applications of Fourier transformations and convolutions are deeply rooted in optics and acoustics, providing the foundation for Fraunhofer diffraction of scalar waves. In optics, convolution in Fourier space is interchangeable with multiplication in diffraction space.[29] It applies to acoustics and optics when the scalar approximation is valid. Therefore, there is some advantage in representing the Fourier transformation in terms of frequency (υ) and time (t). The frequency and time version of the Fourier transform is yet another powerful tool that has important applications in optics. In optics, light is expressed as a function of space for diffraction and with a Fourier transformation, the function of space becomes a function of frequency. The Fourier transform provides a way to describe the content of the plane waves. For example, in optics the Fourier transform represents a light signal as a sum of plane waves. The action of a lens is to focus each plane wave to a diffraction limited spot in a plane one focal length beyond the lens. The intensity pattern in the back of the focal plane is proportional to the absolute square of the two-dimensional Fourier transform of the radiation in the object plane, called the transform plane.

Figure 4 depicts light diffracted from a transparency that is positioned in front of a convergent lens. Each plane wave incident on the lens converges to a point on the back focal plane. Thereafter, the intensity pattern on the back focal plane contains information about the decomposition of the illumination and breaks the light into its wave components or Fourier components.[30]

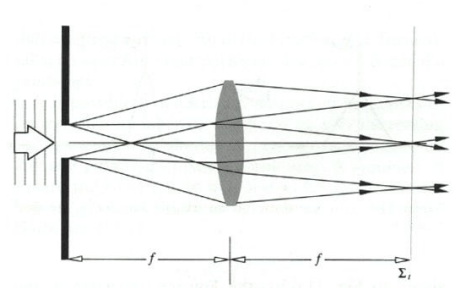


Figure 4

[28] Prof. Tankersley

[29] Steward 87.

[30] Hecht 477.

Figure 5 shows a source emanating light equally from a central point, also called a point source. The point source sends parallel rays of light into a lens located a focal length (f) away. The light undergoes a Fourier transform and is distributed on a screen a focal length away from the lens. The transform changes the point source into irradiance squared with a far-field diffraction pattern. "The two-dimensional Fourier transform is the decomposition of the light into plane-wave components, and each of these components maps to a point on the focal plane of the lens."[31] The mapping is the square of the amplitude of the incoming light. The light enters the lens with a specific amount of energy and undergoes a Fourier transform in which no energy is lost.

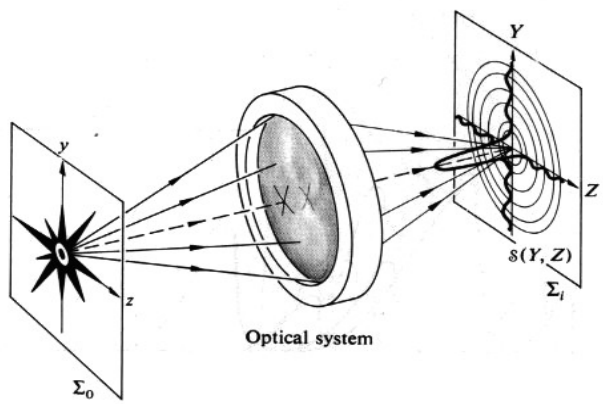
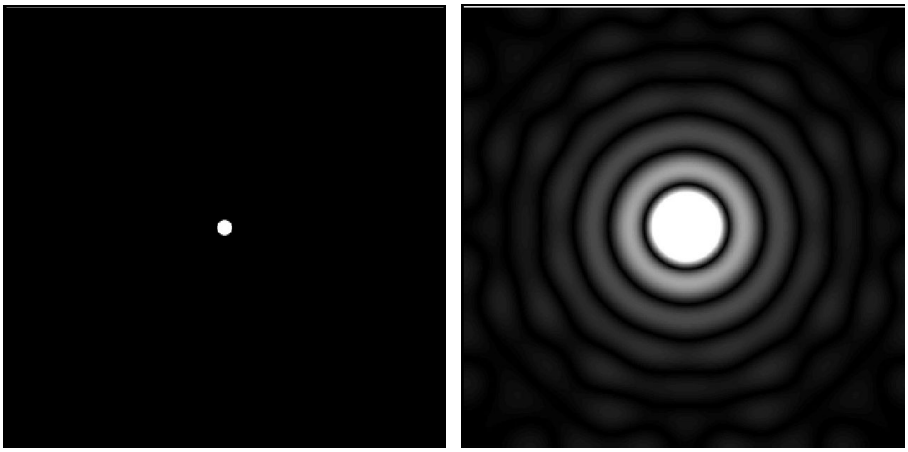


Figure 5[32]

[31] Prof. Tankersley

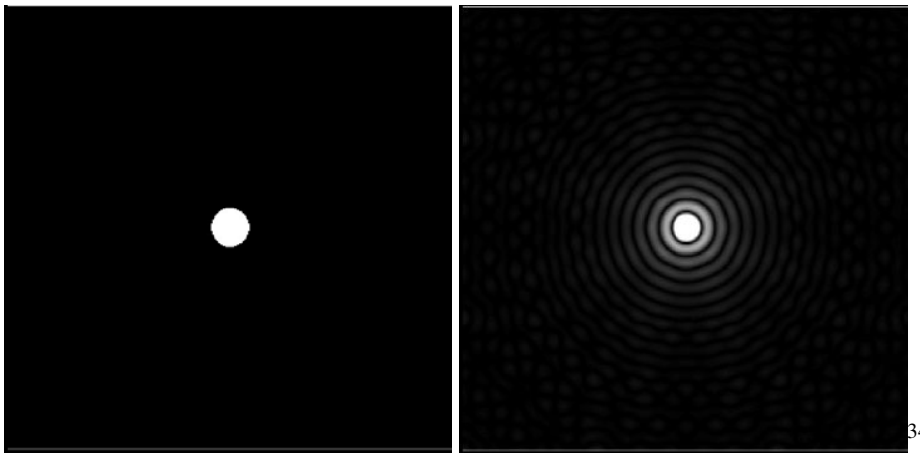
[32] Hecht 485.

Examples of Fourier transforms in optical systems:



[33]

Here, there is a circular aperture used in which light undergoes a Fourier transform, thus creating the Airy pattern.

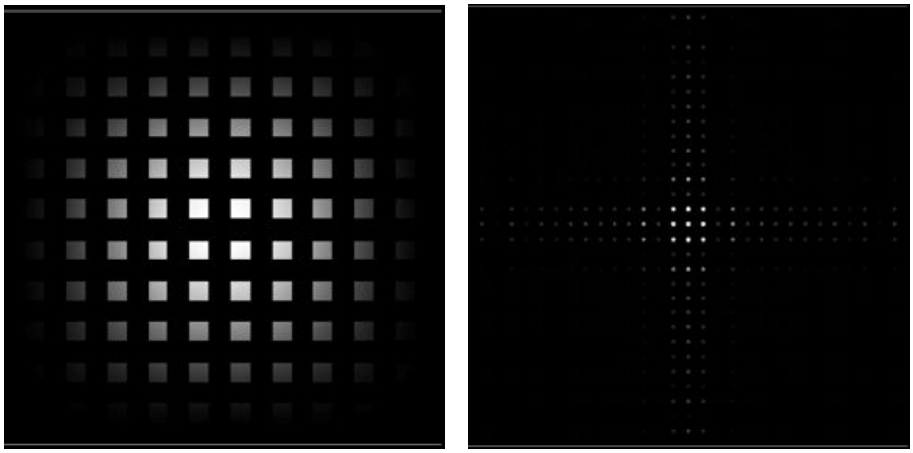


[34]

A similar circular aperture is used again, but this time the diameter is increased. The resultant pattern is a smaller Airy pattern. This pattern is a direct result of the scaling property proven earlier. As the diameter increases, the resultant pattern will decrease in diameter.

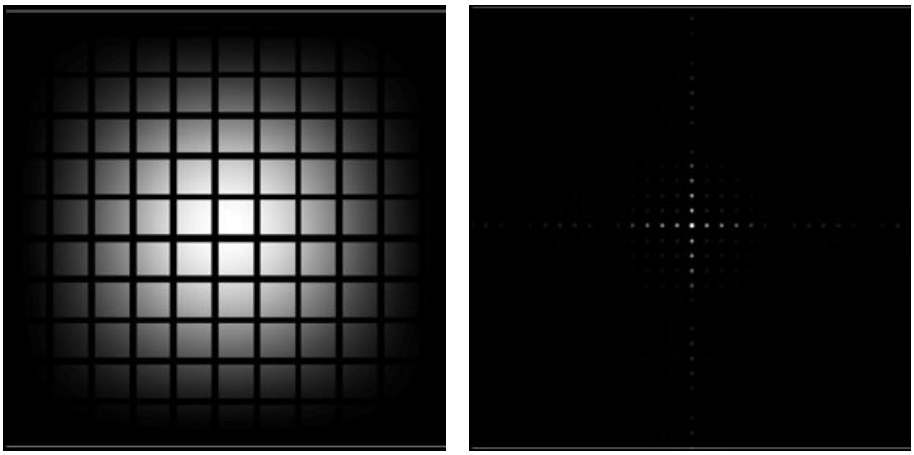
[33] Betzler 7.

[34] Betzler 7.



[35]

Here, we see that larger sub-apertures create smaller envelopes (visible bright spots) with the square of the intensity largest near the center.



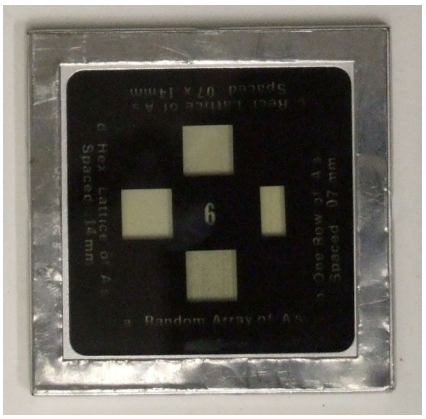
[36]

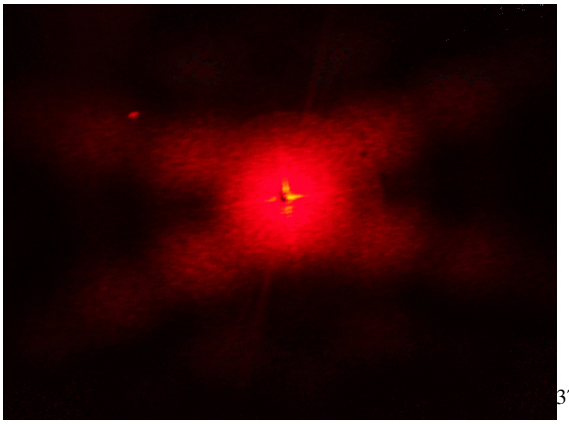
Even larger sub-apertures were used and just as expected, smaller envelopes appear. The square of the intensity is again largest near the center.

[35] Betzler 8.

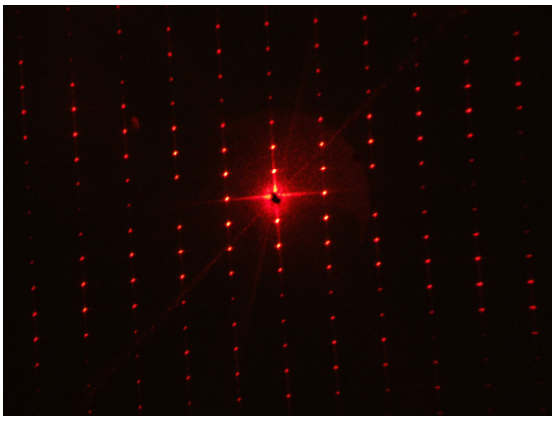
[36] Betzler 8.

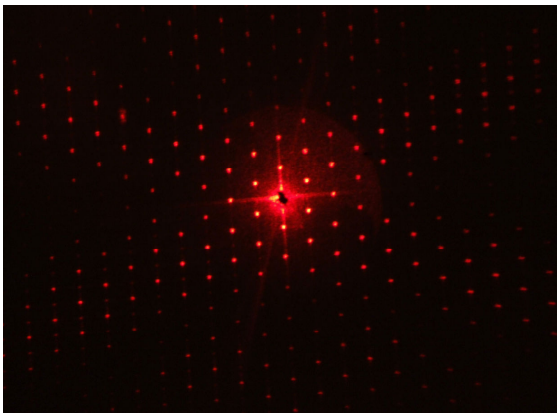
Lab Results:



 [37]

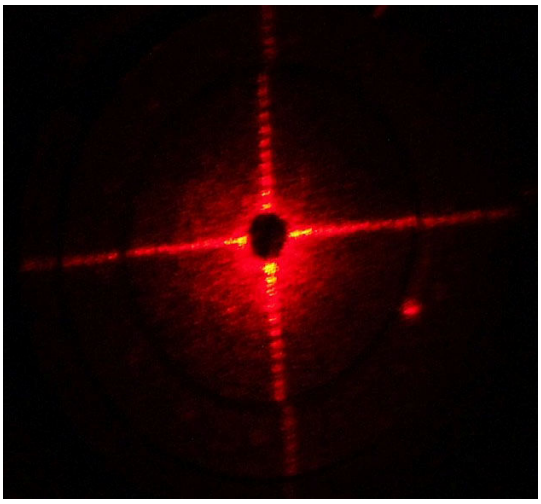
The above optical crystal has differing array of A's. The picture on the right side depicts a random array of A's. Notice the intensity pattern because it will become more evident in following arrays of A's.

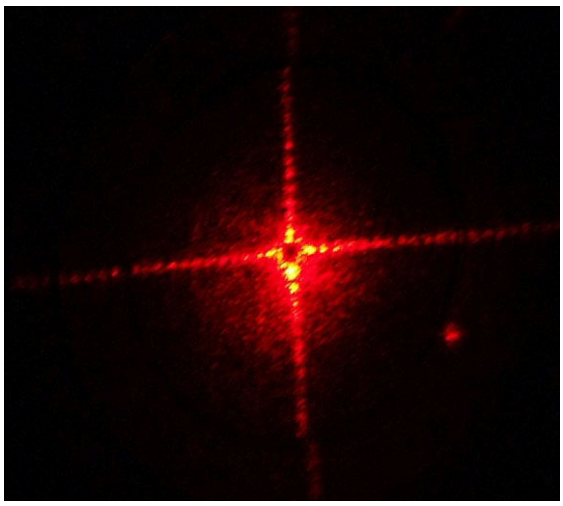




The picture on the left shows the resultant pattern of light through a rectangular array of A's. Notice, as with the random array of A's, that the intensity pattern allows some areas to be more visible than others. The right hand side depicts a hexagonal array of A's. Here we see a different intensity pattern because of changes in the convolution.

[37] All lab photos courtesy of Prof. Tankersley and Walt Teague





If we were to cut high frequencies out in both horizontal and vertical directions, we would see a blurring effect because high frequencies are associated with rapid changes. In the above pictures, we attempted to remove the high frequencies by using a stop, but the photographs were unable to capture the true blurring of the image. However, if we were to block the low spatial frequencies, but allowed the high frequency components to pass, we would ultimately create an edge enhancer.

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