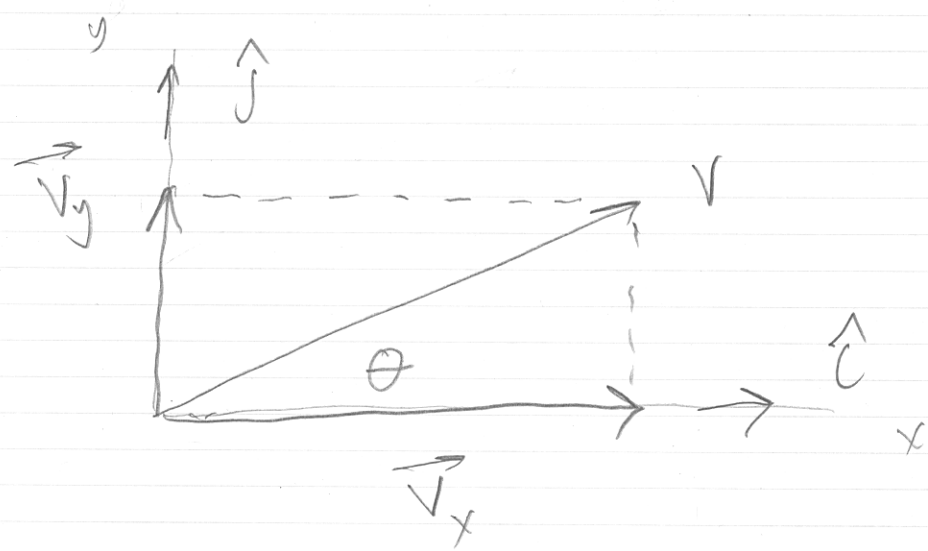
C. Vector Representation in Cartesian Coordinates

The prior graphical technique is instructive but falls short.

→ Computation

Therefore a component approach is used.

Two Dimensions Representation



$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

$\hat{\imath}, \hat{\jmath}$ - unit vectors in the x, y direction

Then

$$\vec{V} = |\vec{V}_x|\hat{\imath} + |\vec{V}_y|\hat{\jmath}$$

or $$\boxed{\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}}$$

<u>Unit Vector</u>

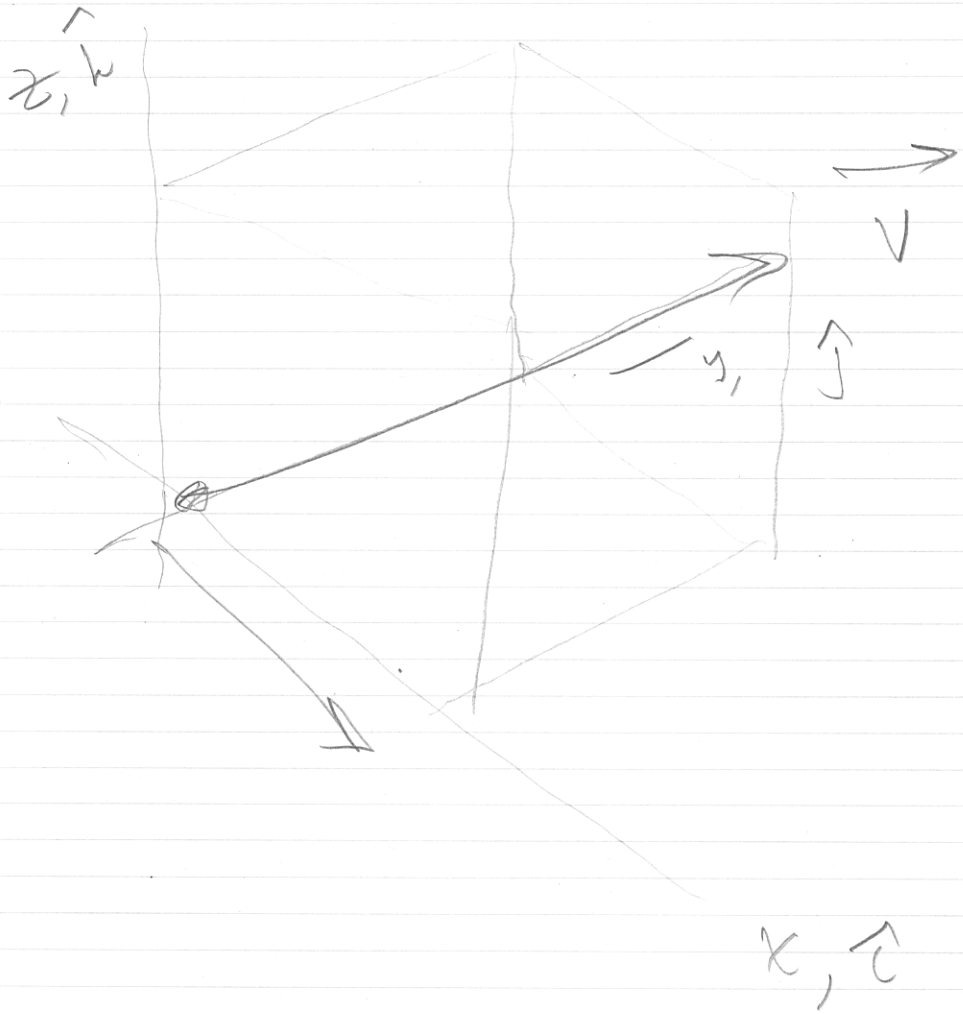
$$\hat{e} = \frac{\vec{V}}{|\vec{V}|} \qquad |\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

$$\boxed{\hat{e} = \frac{\vec{V}}{\sqrt{V_x^2 + V_y^2}}}$$

→ A vector with a fixed direction

→ Often topped with a carat $\hat{e}$

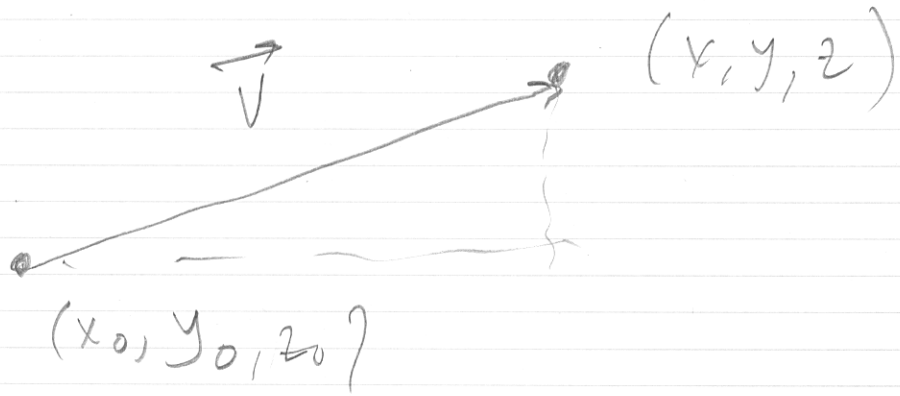
Three Dimensional Representation



$$\boxed{\vec{V} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}}$$

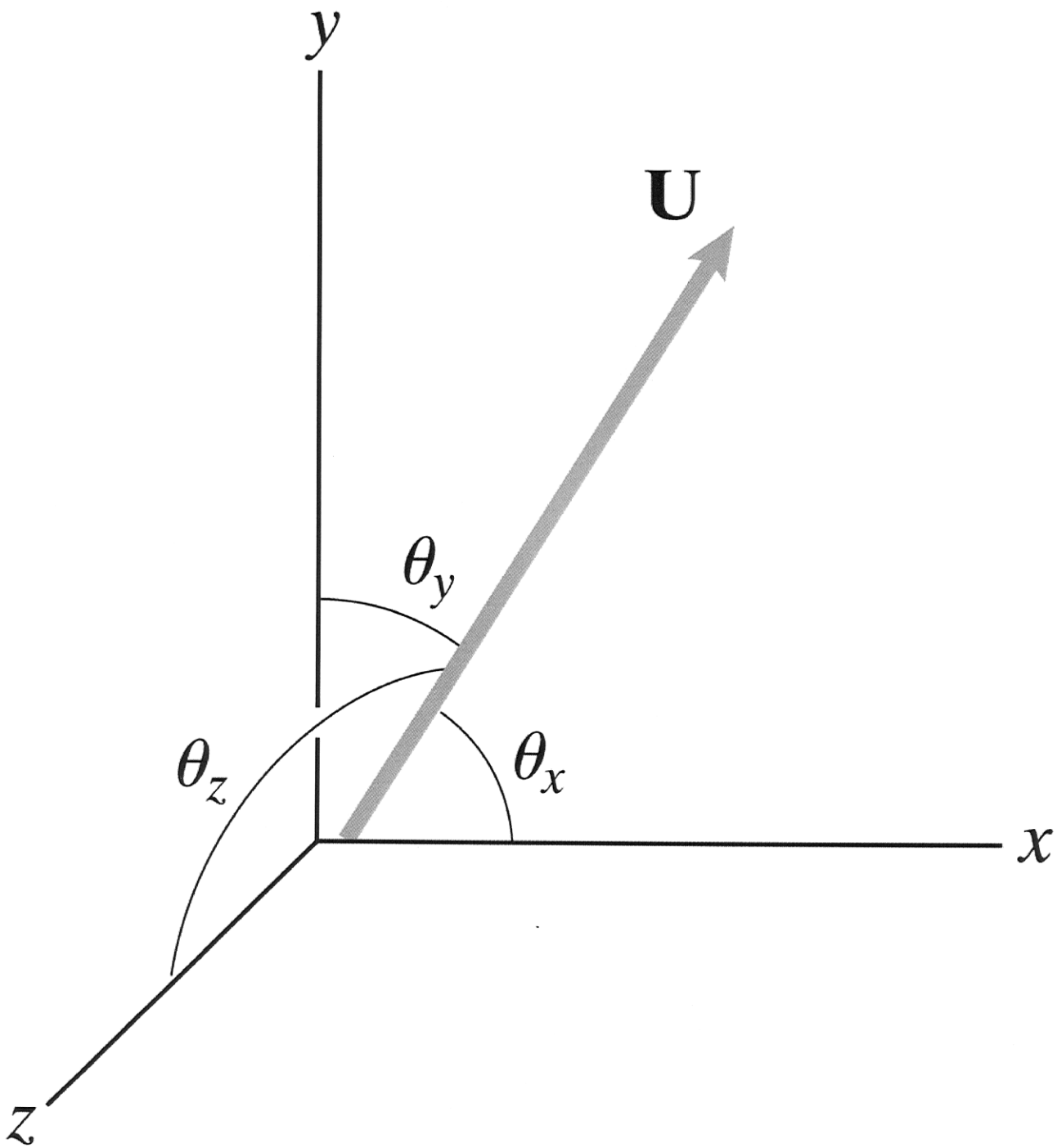
$$\boxed{V^2 = V_x^2 + V_y^2 + V_z^2}$$

Vector Components from Endpoints



$$\boxed{\vec{V} = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}}$$

21.1



Direction Cosines

→ In 2D we can define the direction of a vector using one angle, θ, between vector and a positive coordinate axis.

→ In 3D we need 3, θ_x, θ_y, θ_z referenced to the positive coordinate axes

* NOT with respect to the planes *

FIGURE

$$U_x = U\cos\theta_x = U\,l$$

$$U_y = U\cos\theta_y = U\,m$$

$$U_z = U\cos\theta_z = U\,n$$

$$\boxed{\begin{gathered}\hat{e} = l\hat{i} + m\hat{j} + n\hat{k} \\ l = \cos\theta_x \quad m = \cos\theta_y \quad n = \cos\theta_z\end{gathered}}$$

Right Handed Coordinate Systems

If the fingers of the right hand are pointed in the direction of the positive axis and then bent, as to make a fist, toward the y-axis the thumb points in the direction of the positive z-axis.