~~Continue to build vector algebra~~

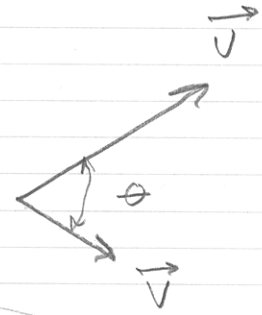
~~→ Big arguements about vector algebra during early 1800's~~

~~→ Gibbs, Hamilton . . .~~

C. Dot Product

Definition

$$\boxed{\vec{U}\cdot\vec{V} = |\vec{U}||\vec{V}|\cos\theta}$$



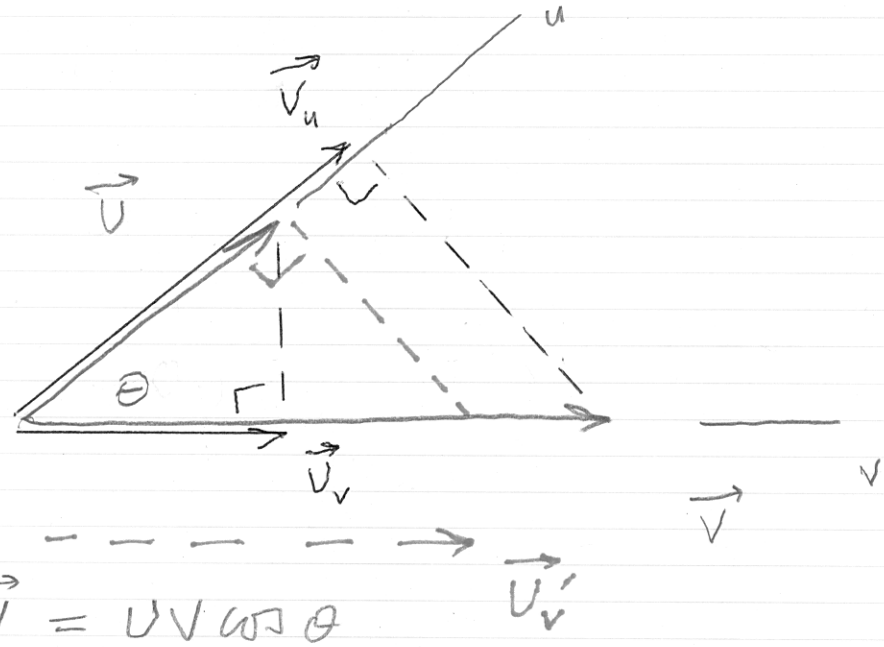
$\vec{U}\cdot\vec{V} = \vec{V}\cdot\vec{U}$ - commutes

$a(\vec{U}\cdot\vec{V}) = (a\vec{U})\cdot\vec{V} = \vec{U}\cdot(a\vec{V})$ - associative w/ scalar multiplication

$\vec{U}\cdot(\vec{V}+\vec{W}) = \vec{U}\cdot\vec{V} + \vec{U}\cdot\vec{W}$ - distributes

Algebraic Properties

Intepretation



$$\vec{u}\cdot\vec{v} = uv\cos\theta$$

$$\vec{u}\cdot\vec{v} = u(v\cos\theta) = uv_u$$

$$\vec{u}\cdot\vec{v} = (u\cos\theta)v = u_v v$$

u_v – component of $\vec{u}$ parallel to $\vec{v}$

v_u – component of $\vec{v}$ parallel to $\vec{u}$

$$\boxed{\vec{u}\cdot\vec{v} = uv_u = u_v v}$$

? Why isn't $\vec{u}_v'$ the right vector

The components of a vector must sum to the original vector

$$\vec{U} = \vec{U}_u + \vec{U}_v$$

therefore each of the components must be smaller than the original vector.

→ The right angle must be opposite the original vector

Component Form

$$\vec{U}\cdot\vec{V} = (U_x\hat{i} + U_y\hat{j} + U_z\hat{k})\cdot(V_x\hat{i} + V_y\hat{j} + V_z\hat{k})$$

$$= U_xV_x\,\hat{i}\cdot\hat{i} + U_xV_y\,\hat{i}\cdot\hat{j} + U_xV_z\,\hat{i}\cdot\hat{k}$$

$$+ U_yV_x\,\hat{j}\cdot\hat{i} + U_yV_y\,\hat{j}\cdot\hat{j} + U_yV_z\,\hat{j}\cdot\hat{k}$$

$$+ U_zV_x\,\hat{k}\cdot\hat{i} + U_zV_y\,\hat{k}\cdot\hat{j} + U_zV_z\,\hat{k}\cdot\hat{k}$$

But

$$\hat{i}\cdot\hat{i} = |1||1|\cos\theta = \cos 0 = 1$$

$$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1}$$

$$\hat{i}\cdot\hat{j} = |1||1|\cos\theta = \cos 90 = 0$$

$$\boxed{\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k}}$$

$$\boxed{\vec{U}\cdot\vec{V} = U_xV_x + U_yV_y + U_zV_z}$$

* Scalar: Vector $\longrightarrow$ Scalar