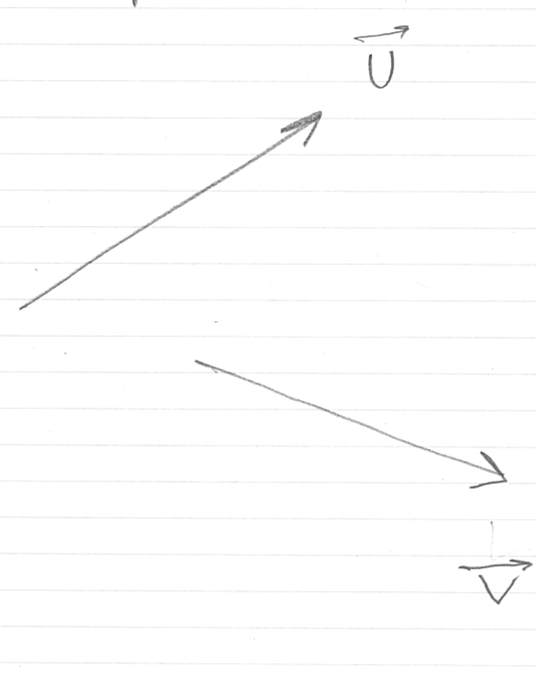
Also

$$\cos\theta = \frac{\vec{U}\cdot\vec{V}}{|\vec{U}||\vec{V}|}$$

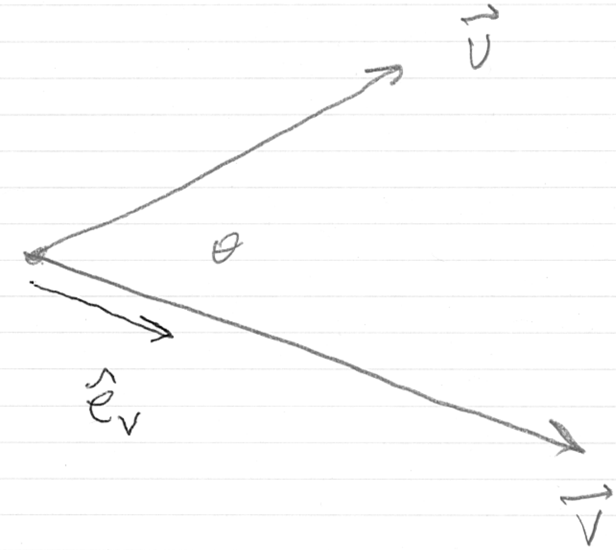
$$\cos\theta = \frac{U_x V_x + U_y V_y + U_z V_z}{|\vec{U}||\vec{V}|}$$

B. Vector Component Parallel and Normal to a Line

Parallell Component



What is the component of $\vec{U}$ along (parallel to) $\vec{V}$ ($\vec{U}_p$)



Recall

$$\vec{U} \cdot \vec{V} = UV\cos\theta$$

$\longrightarrow$ Don't want V involved

$\longrightarrow$ $V \longrightarrow 1$

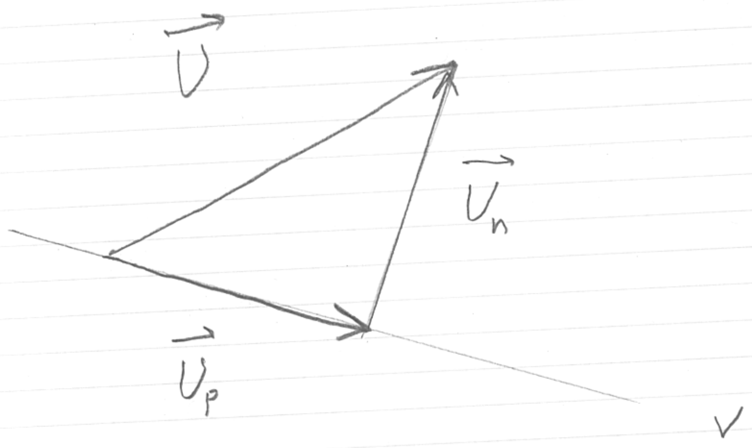
Use a unit vector

$U_p = \hat{e}_V \cdot \vec{U}$ – Magnitude

$\boxed{\vec{U}_p = (\hat{e}_V \cdot \vec{U})\,\hat{e}_V}$ – Vector

Normal Component

From the Triangle Rule



Triangle Rule $\Rightarrow$

$$\vec{U}_p + \vec{U}_n = \vec{U} \Rightarrow \boxed{\vec{U}_n = \vec{U} - \vec{U}_p}$$

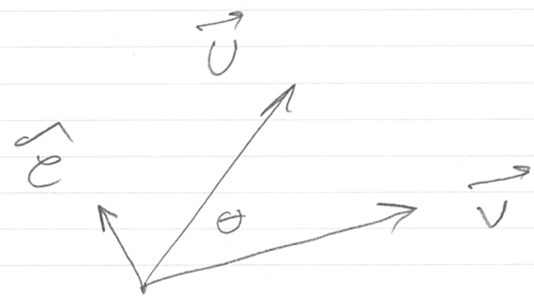
$p \Longrightarrow$ parallel

$n \Longrightarrow$ normal

D. Cross Product

Definition

$$\boxed{\vec{U} \times \vec{V} = |U||V| \sin\theta \, \hat{e}}$$



θ – angle between vectors

$\hat{e}$ – unit vector perpendicular to both $\vec{U}$ & $\vec{V}$

– perpendicular to the plane formed by $\vec{U}$ & $\vec{V}$

– direction determined by right hand rule crossing first vector ($\vec{U}$) into second ($\vec{V}$)

$$\vec{U} \times \vec{V} = -\vec{V} \times \vec{U}$$ – Not commutative

$$a(\vec{U} \times \vec{V}) = a\vec{U} \times V = \vec{U} \times a\vec{V}$$ – associated

$$\vec{U} \times (\vec{V} + \vec{W}) = \vec{U} \times \vec{V} + \vec{U} \times \vec{W}$$ – distributes

<u>Component Form</u>

Unit Vectors

$$\hat{i} \times \hat{i} = 0 \quad \hat{i} \times \hat{j} = \hat{k} \quad \hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{j} \times \hat{j} = 0 \quad \hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \quad k \times \hat{j} = -\hat{i} \quad \hat{k} \times \hat{k} = 0$$

How?

→ Right hand rule

$$\longrightarrow (+)$$

$$\longrightarrow \quad \hat{i} \; \hat{j} \; \hat{k} \; \hat{i} \; \hat{j}$$

$$(-) \longleftarrow$$

$$\vec{U} \times \vec{V} = (U_y V_z - U_z V_y)\hat{i}$$

$$-(U_x V_z - U_z V_x)\hat{j} + (U_x V_y - U_y V_x)\hat{k}$$

or, in terms of a determinant

$$\boxed{\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix}}$$

$$= \hat{i}(U_y V_z - U_z V_y) - (U_x V_z - U_z V_x)\hat{j}$$

$$+ (U_x V_y - U_y V_x)\hat{k}$$

E. Mixed Triple Product

$$\vec{U}\cdot(\vec{V}\times\vec{W})$$

$$\vec{U}\cdot(\vec{V}\times\vec{W}) = \begin{vmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix}$$

$$\vec{V}\cdot(\vec{V}\times\vec{W}) = -\vec{W}\cdot(\vec{V}\times\vec{U})$$

→ Interchanging any two vectors changes the sign

* Interpretation to follow — Chapter 4 *