MEASUREMENT SYSTEMS

Virtually all of the experiments you have done so far in your engineering courses have been static measurements. In this unit we will look at the behavior of measurement systems (transducers) when they are used to measure dynamic (time changing) quantities.

ORDER OF MEASUREMENT SYSTEMS

When we measure dynamic quantities, the behavior of the measurement system is described by a differential equation. Depending upon the basic physical principles used by the measurement system, we need a differential equation of differing order. For example, if the measurement system depends on Newton's Laws of Motion (a bathroom scale is a common day instrument that relies on this principle) then we need a second order differential equation to describe the response of the instrument.

If *x* is the parameter we are measuring (e.g., temperature) and *y* is the output of the measurement system (e.g., the electrical output from a thermocouple), we can define the following:

Zero Order system: $a_0 y = x$

First Order system: $a_1 \frac{dy}{dt} + a_0 y = x$

Second Order system: $a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = x$

Third Order system: $a_3 \frac{d^3y}{dt^3} + a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = x$

and so on

The order of the system relates to the number of *energy stores* in the measurement system. For example, if you use a mercury-in-glass thermometer to measure temperature there is a single energy store (the heat capacity of the mercury) – this device acts predominantly like a 1st order system.

Many devices work on the principle of masses reacting on springs. These devices include the bathroom scale mentioned above, accelerometers, and seismographs. These devices have two energy stores – the spring (elastic potential energy) and the mass (kinetic energy). Thus, these devices respond like 2nd order systems.

There are very few systems that respond like 0th order systems. These are usually fairly simple electronic devices such as a potentiometer.

ZERO ORDER SYSTEMS

We rearrange the zero-order equation as follows:

$$y = Kx$$

where y is the output of the device in response to input x and K is the *static sensitivity*. The units of K are (output)/(input), for example, Volts per degree.

When a zero-order system is subject to a time varying quantity, the output of the device exactly follows the input without any measurement system-generated variations in amplitude and without any time lag (delay) between the input and output.

FIRST ORDER SYSTEMS

First order systems have a single energy store. Examples of such systems include a simple R-C electrical circuit, a mercury in glass thermometer, and a thermocouple. For convenience we will rearrange the first order equation of motion into the form:

$$t\frac{dy}{dt} + y = Kx$$

The symbols mean:

x The time-varying quantity we are measuring – the *measurand*

y The time-varying output from our measurement system. This is the observed quantity

K This is the static sensitivity. The units are (output)/(input), for example, Volts per degree.

t This is the time constant of our measurement system which is determined by the physical characteristics of the system. A measurement system with a small time constant will respond quicker to changes than a system with a large time constant. The time constant depends on the 'size' of the energy store as well as how quickly the energy can get in/out of that store.

For example, a mercury in glass thermometer will respond quicker when plunged into water than it does in air (faster heat transfer with water). A thermometer with a large bead will respond slower than a thermometer with a small bead (bigger energy store).

We will investigate the response of a first order system to three different inputs: step input, ramp input, and sine wave input.

RESPONSE TO STEP INPUT

A step input is defined as:

$$x = 0 \quad t < 0$$
$$x = x_O \quad t \geq 0$$

We solve the first order differential equation in two parts: the *general solution* and the *particular solution*.

We find the general solution by solving the differential equation with the right side of the equation to zero, $Kx = 0$. The general solution for the above first order differential equation is:

$$y_{GEN} = Ce^{-t/t}$$

where *C* is an unknown constant that has to be determined from the initial conditions.

The particular solution for a step input is:

$$y_{PART} = Kx_O$$

The complete solution is the sum of the general and particular solutions, with the constant *C* determined for this specific problem.

$$y = y_{GEN} + y_{PART} = Ce^{-t/t} + Kx_O$$

Initial conditions of y = 0 at t = 0 give:

$$y = Ce^{-t/t} + Kx_O$$
$$0 = C + Kx_O$$
$$C = -Kx_O$$

Hence our final solution is:

$$y = Kx_O\left(1 - e^{-t/t}\right)$$

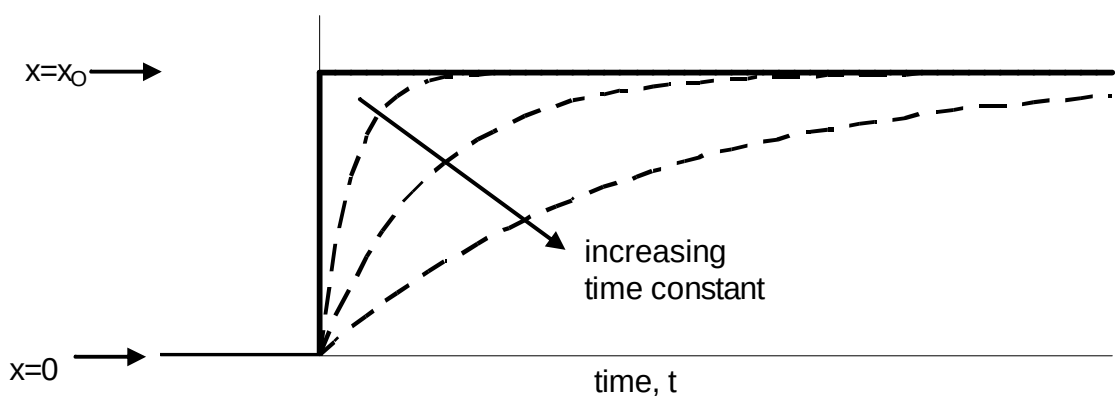
Here is a graph showing the difference between the input (measurand) and output (observed quantity) for three different first order systems responding to a step input. Notice the following:

Even though the output starts to change immediately, a first order system can never respond instantly to an input.

The smaller the time constant, the quicker the system responds. Large time constants lead to slow response.

For a step input, the output is asymptotic to and never quite reaches the input.

There is never an overshoot – the output is always smaller than the input. (Watch for a decreasing step, where the output is always *greater* than the input!)



We could also define a step as starting at non-zero. For example, if the temperature being measured suddenly changes from T_O to T_∞, the temperature measured by a first order measuring system (assuming it is calibrated to read in units of temperature) would be:

$$T = T_\infty - \left(T_\infty - T_O\right)e^{-t/t}$$

or

$$T = T_O + \left(T_\infty - T_O\right)\left(1 - e^{-t/t}\right)$$

or

$$\frac{(T - T_O)}{(T_\infty - T_O)} = \left(1 - e^{-t/t}\right)$$

or

$$\frac{(T_\infty - T)}{(T_\infty - T_O)} = e^{-t/t}$$

EXAMPLE: A first order clinical thermometer is reading 70^OF when it is plunged into a feverish mouth that is at 101^OF. The time constant is $\tau = 60$ seconds. What temperature will the device record at t = 2 minutes?

SOLUTION:

$$\begin{aligned} T &= T_\infty - (T_\infty - T_O)e^{-t/t} \\ T &= 101 - (101 - 70)e^{-120/60} \\ &= 96.805^O F \end{aligned}$$

More typically, you would not know your actual mouth temperature – after all, that is why we are measuring it! Let's change the question a little to make it more realistic.

EXAMPLE: A first order clinical thermometer is reading 70^OF when it is plunged into a feverish mouth. After 2 minutes the recorded temperature was 101^OF. The time constant is $\tau = 60$ seconds. What is the actual temperature of the mouth?

SOLUTION:

$$\begin{aligned} T &= T_\infty - (T_\infty - T_O)e^{-t/t} \\ 101 &= T_\infty - (T_\infty - 70)e^{-120/60} \end{aligned}$$

gives

$$T_\infty = 105.85^O F$$

Quite a difference!

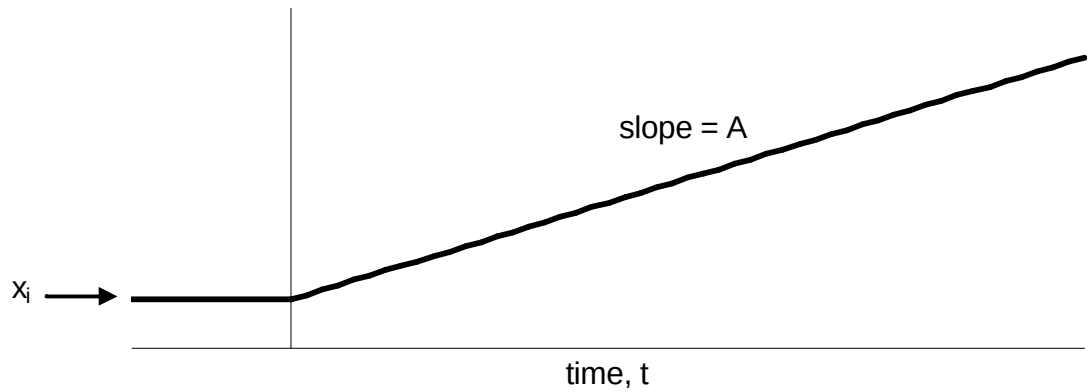
RESPONSE TO RAMP INPUT

A ramp input is defined as:

$$\begin{aligned} x &= x_i & t &< 0 \\ x &= x_i + At & t &\geq 0 \end{aligned}$$

where x_i is the initial value of the measurand, which has not changed for a very long time prior to the start of the ramp, and A is the slope of the ramp input in units of (output) / (time), for example, ^{O}F per minute.

The ramp input is shown in the following figure:



The particular solution for t ≥ 0 is:

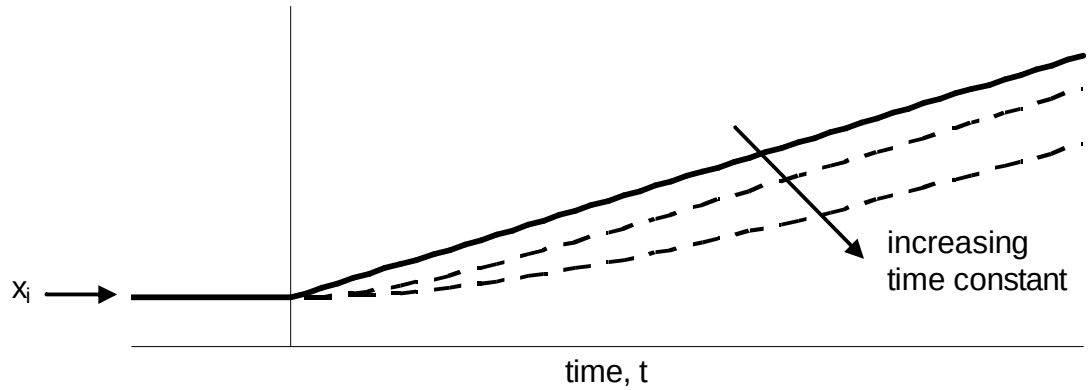
$$y_{PART} = Kx_i + KA\left(t - t\right)$$

The full solution, when solved for the initial conditions, becomes:

$$y = Kx_i + KA\left(t\,e^{-t/t} + t - t\right)$$

Here is a graph showing the difference between the input (measurand) and output (observed quantity) for two different first order systems responding to a ramp input. Notice the following:

> The output does not immediately start to respond to the input (zero slope at t = 0).
>
> The larger the time constant, the more the output lags behind the input.
>
> The output is always smaller than the input. (Watch for a negative slope where the output is always *greater* than the input!)



INSTANTANEOUS ERROR

As experimentalists, we are concerned with the errors introduced by our measurement system. One such error is the instantaneous error. By that, we mean, *"What is the difference between the measurand and observed quantity at any instant in time?"*

Notice the curved shape to the observed quantity just after time t = 0. Usually this transient region is very short in duration compared to the overall measurement. This leads us to notice that after the ramp has been applied for a longer time, the output becomes a straight line, parallel to the input. Therefore, when we estimate the errors introduced by our measurement device, we only consider the straight portion of the results.

Mathematically, we solve the output equation for large *t*:

$$y = Kx_i + KA\left(t\,e^{-t/t} + t - t\right)$$

$$\text{when } t >> 0 \quad y = Kx_i + KA\left(t - t\right)$$

If there were no error, the output would exactly follow the input:

$$x = x_i + At$$

$$y_{NO\,ERROR} = Kx$$

$$y_{NO\,ERROR} = Kx_i + KAt$$

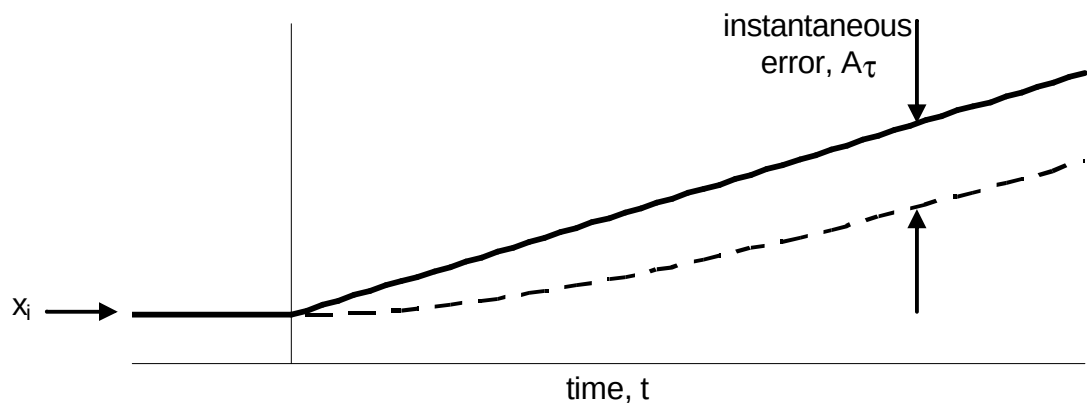
The error is thus:

$$\begin{aligned}\text{error} &= y - y_{NO\,ERROR}\\ &= \left\{Kx_i + KA\left(t - t\right)\right\} - \left\{Kx_i + KAt\right\}\\ &= -KAt\end{aligned}$$

For a measurement system calibrated to read in "correct" units, the overall value for K is one. Thus the error in our measurement system becomes:

$$\text{instantaneous error} = -At$$

The error is demonstrated in the following figure:



TEMPORAL ERROR

We saw that the output was always behind (i.e., lagged) the measurand. This leads to another question, *"How long after the measurand has passed a prescribed level does the measurement system indicate that level?"* This is a temporal (time domain) error.

We calculate the error by looking at a certain level and calculating the difference in time between input and output. Again, we only consider the 'large-time' situation.

If there were no error, the output would exactly follow the input, and as before:

$$y_{NO\ ERROR} = Kx_i + KAt$$

The actual (observed) output is:

$$y = Kx_i + KA\left(t\,e^{-t/t} + t - t\right)$$

$$\text{when } t >> 0 \quad y = Kx_i + KA\left(t - t\right)$$

We ask, what is the time difference for these two levels to be the same?

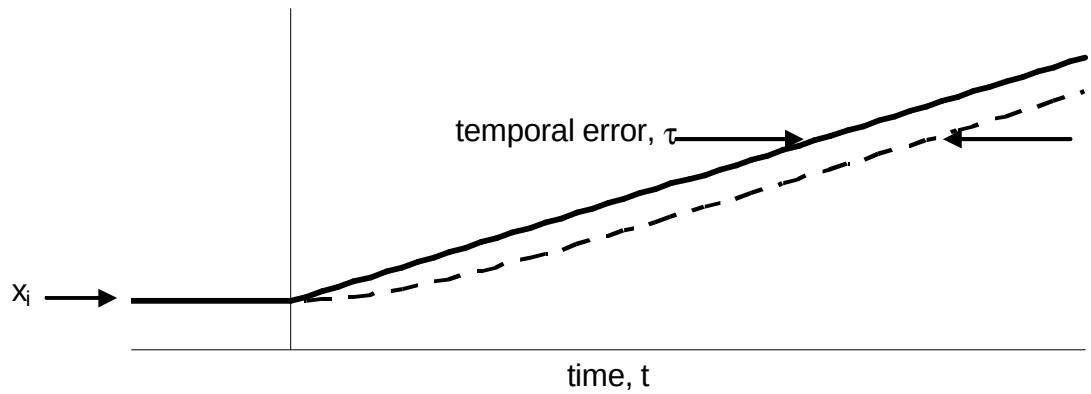
$$y_{NO\ ERROR} = Kx_i + KAt_1$$

$$y_{OBSERVED} = y = Kx_i + KA\left(t_2 - t\right)$$

When $y_{NO\ ERROR} = y$ we solve the equations to get:

$$(t_1 - t_2) = t$$

This error is independent of both K and A. Thus, after the initial transient period, the output of a first order system lags a ramp input by one time constant. The error is demonstrated in the following figure:



EXAMPLE:
The heating system in a very cold house (less than 50°F) is turned on. The heating system is controlled by a thermocouple with a 5-minute time constant. Assume the temperature increases approximately linearly with respect to time. During heating it was noticed that it took 40 minutes for the temperature to rise from 55°F to 71°F.

When the thermocouple indicates 75°F it switches off the heating system. Calculate the following:

a) How long after the actual house temperature passes 75°F does the thermocouple indicate that temperature?

b) At the instant the heating system is switched off, what is the actual house temperature?

SOLUTION:
We assume that we are not concerned with the initial transient behavior of the thermocouple. We are therefore in the straight-line portion of the response curves.

a) The temporal error is the time constant. Therefore, the thermocouple will indicate 75°F some 5 minutes (the time constant, τ) after the actual temperature passes 75°F.

b) The instantaneous error is $-At$. First, calculate the slope of the input. The change was (71 – 55) = 16°F in 40 minutes, so:

$$A = \frac{16}{40} = 0.4^{O}F/\text{minute}$$

The instantaneous error is $-At = -0.4 \times 5 = -2.0^{O}F$. Therefore, the thermocouple reads 2^{O}F below the actual temperature, and the actual room temperature will be 77°F.

EXAMPLE:
After the heating system was turned off in the previous example, the air conditioner was switched on. The air conditioner can reduce the temperature in the house at a rate of 0.6^{O}F/minute. The thermocouple switches the air conditioner off when the temperature passes 70°F. Calculate the actual house temperature when the thermocouple indicates 70°F.

SOLUTION:
The ramp input has a negative slope:

$$A = -0.6^{O}F/\text{minute}$$

The instantaneous error is $-At = -(-0.6) \times 5 = +3.0^{O}F$. Therefore, the thermocouple reads 3°F *above* the actual temperature, and the actual room temperature will be 67°F.

RESPONSE TO HARMONIC INPUT

In engineering we do not very often come across pure harmonic motion. However, when we have complicated periodic functions we often analyze them using Fourier analysis. What this essentially does is to break the complicated waveform (for example, the temperature in the combustion chamber of an internal combustion engine) into a series of harmonic signals. We then separately analyze each of those harmonic signals.

Before we can do this, we need to understand how our measurement systems respond to pure harmonic excitation.

Throughout this section we assume that the harmonic excitation has been going on for a long time. This way, we can ignore the transient effects on our measurement system caused when the harmonic input first starts. That means we ignore the general solution, and only consider the particular solution.

The measurand (input) is:

$$x = x_O \sin(wt)$$

In this equation:

x	is the time-varying quantity we are measuring – the measurand
x_O	is the mean-to-peak amplitude of the harmonic excitation
w	is the circular frequency of the excitation. In this equation the circular frequency <u>must be</u> in units of radians per second (rad/s).
t	is time in seconds

The particular solution is:

$$y = \frac{Kx_O}{\sqrt{1+(wt)^2}} \sin(wt + f)$$

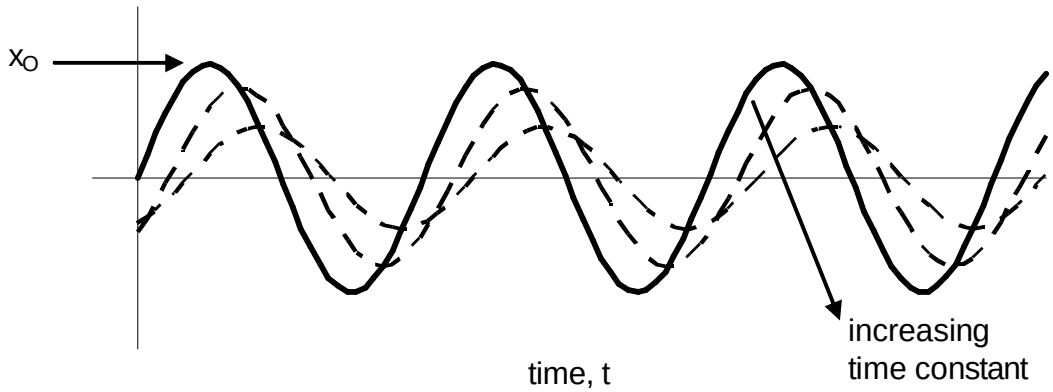
where

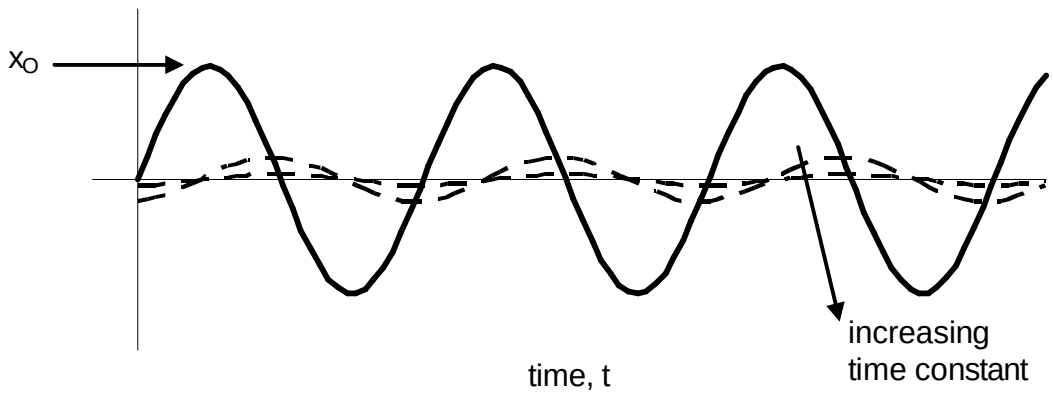
y	is the time-varying output of the measurement system (the observed quantity)
f	is the phase between the input and output, measured in radians. The phase is calculated as the inverse tangent of frequency and system time constant:

$$f = -\tan^{-1}(wt)$$

For the previous sections on step input and ramp input we looked at graphs of input and output versus time. We can do the same for harmonic excitation. Here are two examples.

The first is for a very low frequency of excitation, and the second is for a higher frequency. The time axes have been changed to show the same number of cycles.





Notice the following:

> The output is always behind (lagging) the input.
>
> The magnitude of the output is always smaller than the magnitude of the input.
>
> As the time constant gets bigger, the magnitude of the output gets smaller and the lag gets bigger.
>
> As the frequency goes up, the output of our measurement system gets smaller.

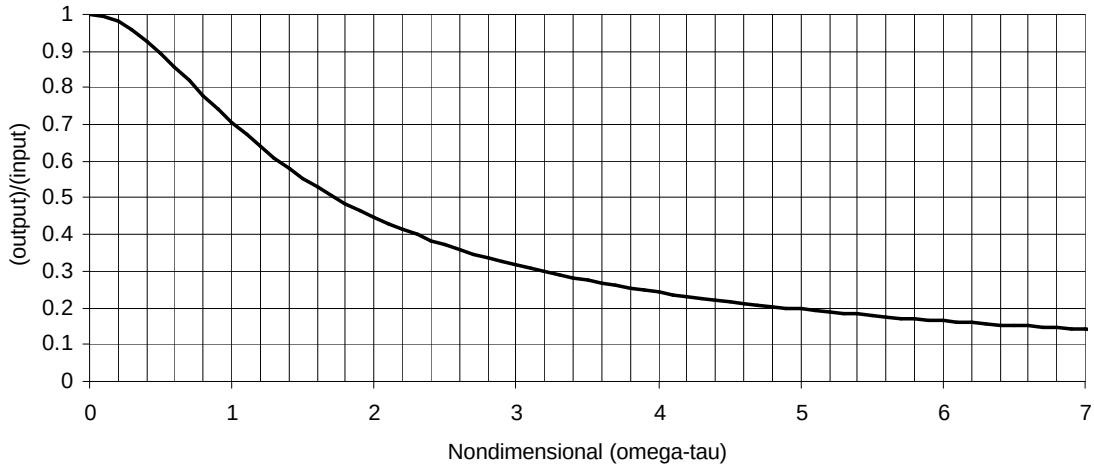
These time-based graphs have very limited use for harmonic excitation. More often we look at the magnitude and phase characteristics as functions of frequency. Actually, rather than 'raw' frequency, we use the nondimensional frequency wt . The frequency w is in units of radians per second and the time constant t is in units of seconds. Therefore the nondimensional frequency has no units.

The following two graphs show how the amplitude and phase of the measurement system change as the excitation (input, measurand) frequency changes. The vertical axes show

the amplitude ratio of (output) / (input) and the horizontal axes shows the nondimensional frequency wt .

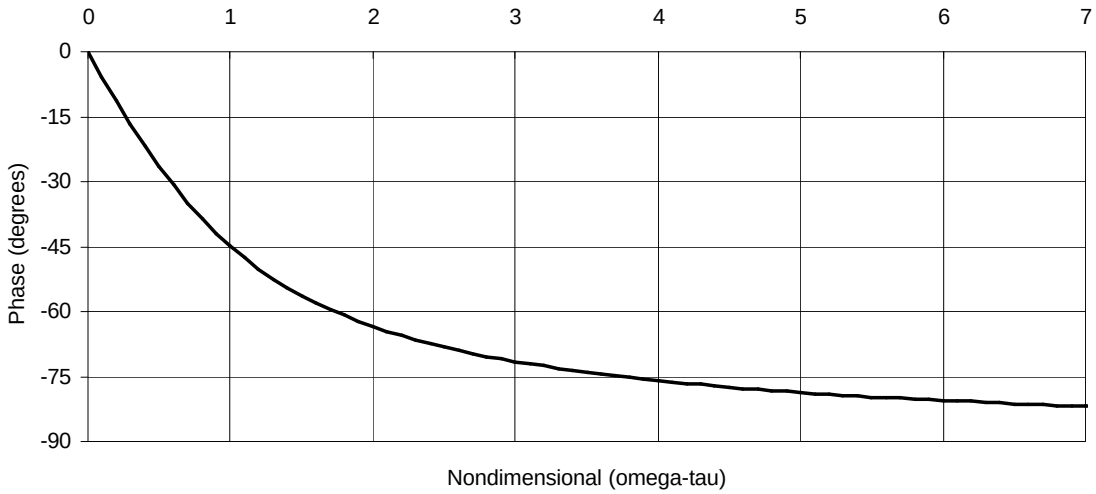
This graph shows the equation:

$$\frac{Y}{Kx_O} = \frac{1}{\sqrt{1+(wt)^2}} \text{ versus } (wt)$$



This graph shows:

$$f = -\tan^{-1}(wt) \text{ versus } (wt)$$



EXAMPLE:

A first order measurement system with a time constant of $\tau = 0.1$ seconds is used to measure a harmonically varying voltage. The voltage has a peak-to-peak amplitude of 6 Volts and a frequency of 5 Hz. Calculate the following:

a) Frequency of the observed signal
b) Mean-to-peak amplitude of the observed signal
c) Phase (in degrees) of the observed signal
d) Time delay (lag) of the observed signal.

SOLUTION (analytical):

a) The input and observed signals both have the same frequency. Therefore, observed frequency = 5 Hz.

b) mean-to-peak amplitude = (peak-to-peak)/2 = 6/2 = 3 V

$$w = 2pf = 2p5 = 31.42 rad/s$$

$$\frac{Y}{Kx_O} = \frac{1}{\sqrt{1+(wt)^2}}$$

hence $Y = \frac{Kx_O}{\sqrt{1+(wt)^2}} = \frac{3}{\sqrt{1+(31.42\times 0.1)^2}} = 0.910V$ (observed mean-to-peak)

c) $f = -\tan^{-1}(wt) = -\tan^{-1}(31.42\times 0.1) = -72.3^O$

d) One cycle takes 1/5 = 0.2 seconds. Therefore 72.3^O takes $0.2\times\frac{72.3^O}{360^O} = 0.0402s$

SOLUTION (graphical):

a) Same solution as analytical solution (5 Hz)

b) mean-to-peak amplitude = (peak-to-peak)/2 = 6/2 = 3 V

$wt = 2pft = 2p5\times 0.1 = 3.142$ (no units)

from the graph, at $wt = 3.142$ we get $\frac{Y}{Kx_O} = 0.3$ (approx) so $Y = 0.3\times 3 = 0.9V$

c) from the graph, at $wt = 3.142$ we get $f = -70^O$ (approx)

d) Same solution as analytical solution (0.0402s)

EXAMPLE:
A first order system measures a harmonic voltage and indicates the mean-to-peak amplitude is 5 Volts. If the measurement system has a time constant of $\tau = 0.3$ seconds and the signal has a frequency of 10 Hz, what is the mean-to-peak amplitude of the measurand?

SOLUTION:

$$wt = 2pft = 2p10 \times 0.3 = 18.85$$

$$\frac{Y}{Kx_O} = \frac{1}{\sqrt{1+(wt)^2}}$$

hence $Kx_O = Y\sqrt{1+(wt)^2} = 5\sqrt{1+(18.85)^2} = 94.4V$ (mean-to-peak)

Clearly, using a measurement system with an inappropriate time constant can lead to large measurement errors.

Special Situation:
One situation warrants further investigation. Very often the signal we are considering consists of a harmonic signal with a DC level offset. For example, if we are measuring temperature in an engine, the temperature cannot go negative – there is a periodic signal with a DC offset. The equation for the waveform we consider is:

$$x = \bar{x} + x_O \sin(wt)$$

The output from a first order system will be:

$$y = K\bar{x} + \frac{Kx_O}{\sqrt{1+(wt)^2}} \sin(wt + f)$$

What this means is that the output from our first order system will have the correct DC offset, superimposed with the harmonic content determined using the previous harmonic excitation work in this handout.

Some observations and questions:

> If you wish to measure the DC component, should you use a first order system with a short or long time constant?
>
> What if you need to measure the harmonic part?

EXAMPLE: You are designing a thermocouple first order system to monitor the temperature in a passageway. The system will activate a heater and/or air conditioner if the observed temperature varies by more than 0.2^{O}F from the required value of 72°F. It has been observed (using high-speed very accurate equipment) that the movement of hot equipment on carts past the thermocouple causes a heating effect such that the actual temperature is approximately harmonic, following the equation:

$$T = 72 + 0.4\sin(0.005t)\ {}^{O}F$$

a) What is the shortest permissible time constant for the thermocouple such that it will not activate the heater/air conditioner just because of the cart activity?

b) Someone opens a window and the temperature suddenly increases by 5°F. Using the time constant you found in a) above, what will be the time delay before the thermocouple system activates the air conditioner?

SOLUTION:

a) We need the observed mean-to-peak amplitude to be less than 0.2^{O}F. We have

$$\frac{Y}{Kx_O} = \frac{1}{\sqrt{1+(wt)^2}}$$

$$\frac{0.2}{0.4} = \frac{1}{\sqrt{1+(0.005t)^2}}$$

$$t = 346.4s$$

A system with a time constant of 346.4s or more will be OK. Let's choose τ = 360 seconds.

b) We now have a step input (back to the top of this handout!). The response of the system is given by:

$$T = T_O + (T_\infty - T_O)\left(1 - e^{-t/t}\right)$$

Putting in the values and solving for time, t:

$$72.2 = 72 + (77 - 72)\left(1 - e^{-t/360}\right)$$

$$t = 14.7s$$

Note that we have managed to get a control system to respond quickly (less than 15 seconds) to a small increase in temperature, while 'ignoring' the cart-induced fluctuations.

FOLLOW-ON QUESTION: How would your solution be different if you needed to measure the temperature fluctuations caused by the passing equipment?

THERMOCOUPLES

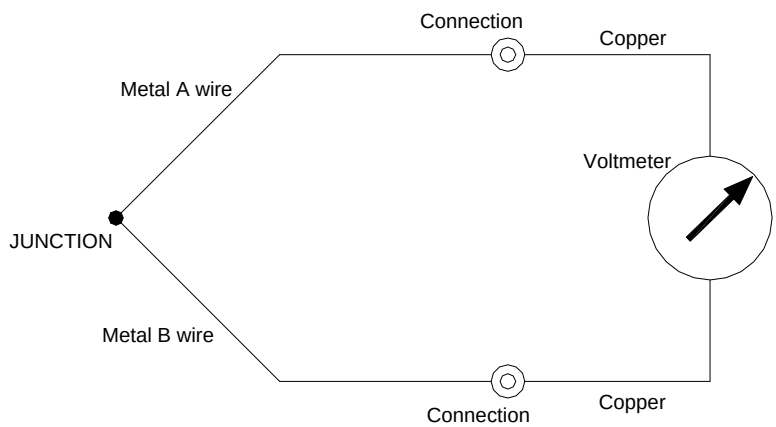
A thermocouple is an electrical device that is commonly used to measure temperature. If any two dissimilar metals are connected together a voltage is developed. This phenomenon is called the Seebeck effect (Thomas Seebeck, 1821). The level of the voltage depends on the temperature of the connection. By measuring the induced voltage, we can get a measure of the temperature of the junction.

Since there is only one "energy sink" (the heat capacity of the metal forming the junction) a thermocouple acts like a first order system.

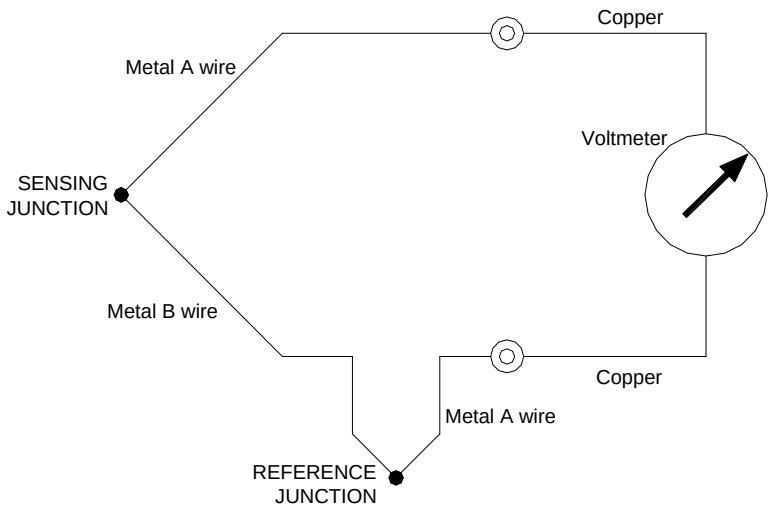
Note the following:

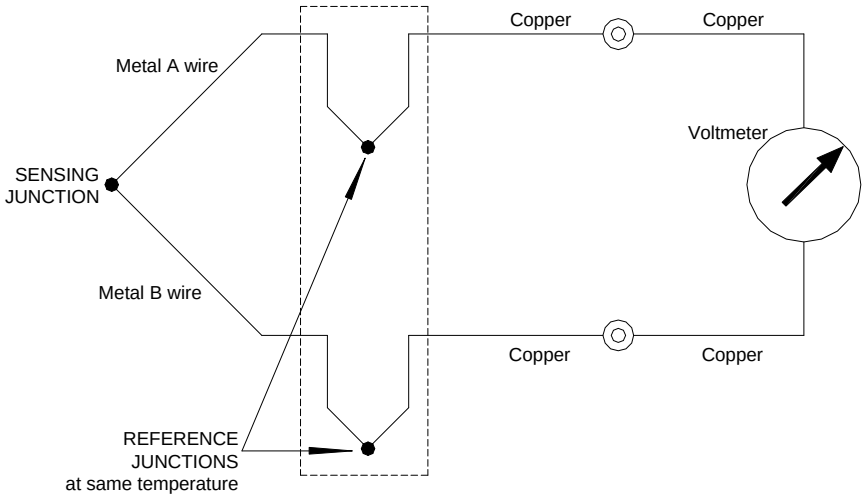
The voltage must be made with no current flow. Therefore we need to use a very high input impedance voltmeter (DVM)

Once we connect the two wires to the DVM, we have introduced 2 more connections, so there are now 3 connections with the Seebeck effect.



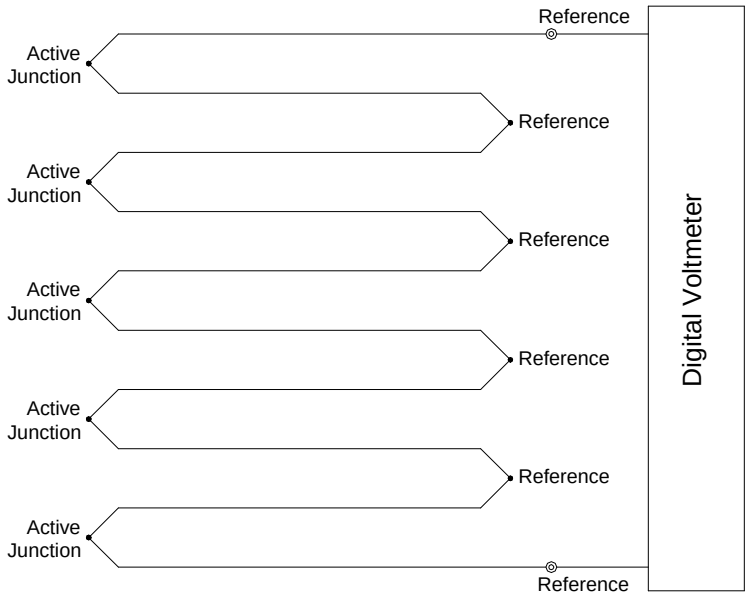
How can we get round the problem of the voltage caused by the DVM connections? We do this by introducing one or 2 more connections, called the *reference junction.* The following figures show two ways of making this circuit. Note that the second figure uses copper from the reference junction to the DVM. What are the advantages of this method?



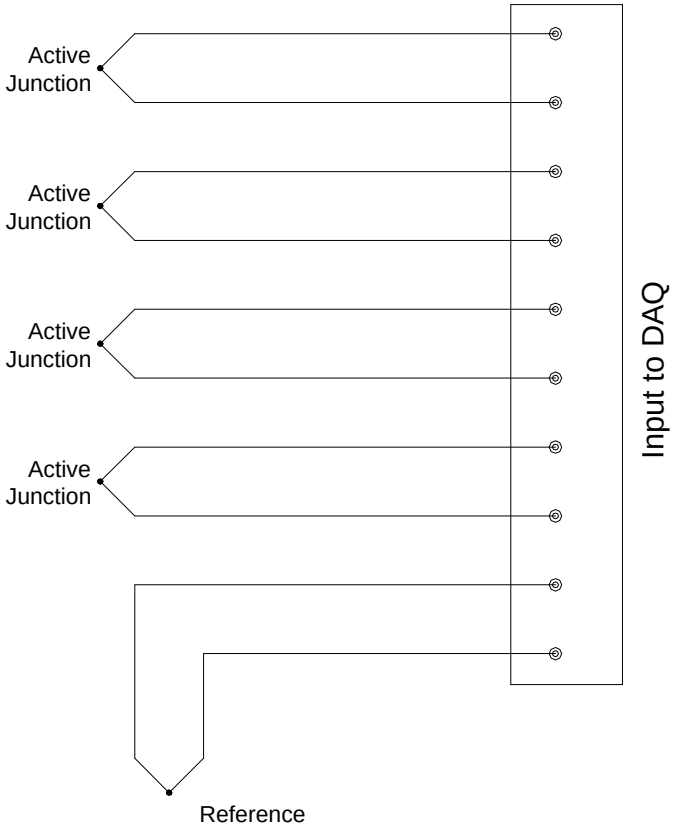


The reference junction is held at a constant temperature, often $0^{O}C$ – melting ice. There are many electronic devices that simulate this temperature without the need for ice.

The following figures show two more ways of connecting thermocouple wires. The thermopile connects several thermocouples together and gives a voltage output greater than a single thermocouple. The DAQ arrangement shows how it is only necessary to have a single reference junction when measuring temperatures with several thermocouples. In this configuration the DAQ software determines the difference in voltage between each thermocouple and the reference junction. It then converts these voltages to temperatures.



Thermopile



Four thermocouples and a single reference junction used for a Data Acquisition (DAQ) system

The level of voltage generated by the wire junction depends on the type of metals in the two separate wires. NIST (National Institute of Standards and Technology) restricts the number of different wires that can be used for commercial thermocouples, and has issued standard calibration curves. Unfortunately the voltage generated is not a linear function of temperature. Therefore the voltage has to be determined using a lookup table. In a computer application the curves are usually determined as polynomial functions.

The standard thermocouple materials are shown in the following table. The sensitivity is only an approximate value to help in selection. Exact calculations of temperature must be done using the table of thermocouple outputs.

Thermocouple Type	Materials	Lead wire color	Operating range (^{O}C)	Sensitivity ($mV/^{O}C$)
T	Copper & constantan	Blue	-250 to +400	0.052
E	Chromel & constantan	Purple	-270 to +1000	0.076
J	Iron & constantan	Black	-210 to +760	0.050
K	Chromel & alumel	Yellow	-270 to +1372	0.039
R	Platinum & platinum/13% rhodium	Green	-50 to +1768	0.011
S	Platinum & platinum/10% rhodium	Green	-50 to +1768	0.012
C	Tungsten/ 5% rhenium & tungsten/ 26% rhenium	White, red tracer	0 to +2320	0.020

The following table is to be used for all thermocouple problems on this course. The table gives the output (in mV) of different thermocouples assuming the reference junction is held at $0^{O}C$.

Temperature (^{O}C)	T-Type	E-Type	J-Type	K-Type	R-Type	S-Type
-250	-6.181	-9.719		-6.404		
-200	-5.603	-8.824	-7.890	-5.891		
-150	-4.648	-7.279	-6.499	-4.912		
-100	-3.378	-5.237	-4.632	-3.553		
-50	-1.819	-2.787	-2.431	-1.889		
0	0.000	0.000	0.000	0.000	0.000	0.000
20	0.789	1.192	1.019	0.798	0.111	0.113
40	1.611	2.419	2.058	1.611	0.232	0.235
60	2.467	3.683	3.115	2.436	0.363	0.365
80	3.357	4.983	4.186	3.266	0.501	0.502
100	4.277	6.317	5.268	4.095	0.647	0.645
120	5.227	7.683	6.359	4.919	0.800	0.795
140	6.204	9.078	7.457	5.733	0.959	0.950
160	7.207	10.501	8.560	6.539	1.124	1.109
180	8.235	11.949	9.667	7.338	1.294	1.273
200	9.286	13.419	10.777	8.137	1.468	1.440
220	10.360	14.909	11.887	8.938	1.647	1.611
240	11.546	16.417	12.998	9.745	1.830	1.785
260	12.572	17.942	14.108	10.560	2.017	1.962
280	13.707	19.481	15.217	11.381	2.207	2.141
300	14.860	21.033	16.325	12.207	2.400	2.323
350	17.816	24.961	19.089	14.292	2.896	2.786
400	20.869	28.943	21.846	16.395	3.407	3.260
450		32.960	24.607	18.513	3.933	3.743
500		36.999	27.388	20.640	4.471	4.234
600		45.085	33.096	24.902	5.582	5.237
700		53.110	39.130	29.218	6.741	6.274
800		61022		33.277	7.949	7.345
900		68.873		37.235	9.203	8.448
1000		76.358		41.269	10.503	9.585

Example:
A pair of K-type thermocouples has a voltage difference of 17.893 mV. The cooler junction is known to be at 300°C. What is the temperature of the hot junction?

Solution:
All calculations must be done in voltages. This is because the thermocouple calibration is not quite linear. The equation is:

$$V_{OUT} = V_{ACTIVE} - V_{REFERENCE}$$

so

$$17.893mV = V_{ACTIVE} - V_{REFERENCE}$$

We need convert the 300°C to mV by looking up in the table to get $V_{REFERENCE} = 12.207mV$. The calculation now is:

$$17.893mV = V_{ACTIVE} - 12.207$$
$$V_{ACTIVE} = 30.100mV$$

Interpolation in the table gives the temperature for 30.100mV as 721.7 ^{O}C.

Example:
The hot junction of an R-type thermocouple pair is at a known temperature of 900 ^{O}C. The cold junction is at 180 ^{O}C. If the DAQ system is calibrated assuming the reference junction is held at 0 ^{O}C, what will be the indicated temperature?

Solution:
Look up the temperatures in the table to get voltages:

180 ^{O}C (reference)	1.294 mV
900 ^{O}C (active)	9.203 mV

$$V_{OUT} = V_{ACTIVE} - V_{REFERENCE}$$
$$= 9.203 - 1.294 = 7.909mV$$

Interpolation in the table gives the temperature for 7.909 mV as 796.7^{O}C.

Note that if you “just” do the calculations as temperature calculations, you will get the wrong answer (in this case, 900-180 = 720°C).

PROBLEMS

1stOrder-1:
A thermocouple is subject to a step change in temperature. Determine the time it will take the thermocouple to record 99% of this change for each of the following thermocouple time constants:

τ = 0.1s, 1s, 10s, and 1 min

1stOrder-2:
A fluid has a sinusoidal temperature variation of $\pm 2^{O}$F with a time period of 10 seconds. For each of the time constants in problem 1, calculate:

a) The indicated (i.e., mean-to-peak) output from the systems.
b) The phase lag in degrees.

1stOrder-3:
The following data were obtained by plunging a thermocouple into a mixture of ice and water at a known temperature of 0°C. The thermocouple was initially at the ambient room temperature of 20°C.

Time (s)	0.1	0.5	1.0	2.0	3.0
Temp (^{O}C)	16.7	8.1	3.3	0.6	0.1

a) Create a worksheet in MathCAD that plots the raw time vs. temperature data
b) Identify a linearizing transformation for the expected response of the system
c) Transform the data, then do a complete linear regression of the data
d) Use the results from c) to determine the time constant of the thermocouple.
e) In the same worksheet re-plot the original temperature vs. time data and overlay the theoretical response determined using your calculated time constant.

1stOrder-4:
A pair of K-type thermocouples is used to measure the temperature in a hot air duct. The DAQ is calibrated assuming the reference junction is held at 0°C and the indicated temperature is 120OC. The reference junction is actually at 80°C. What is the actual hot air duct temperature?

1stOrder-5:
The same system (DAQ and reference junction) as in problem 4 is now used to control a freezer. It is required that the freezer is maintained at -50°C. What temperature should the DAQ indicate?