INTRODUCTION TO MATHCAD
Solving Equations

Wouldn't it be nice if you could easily get your computer to solve all kinds of equations? That is, solving a **single equation**, or solving **simultaneous linear equations**, or even solving **simultaneous nonlinear equations.**

Well, now you can! This lesson shows you some different ways of doing just that.

Open a new worksheet in preparation for this lesson, and format it as an enclosure.

SOLVING SINGLE ALGEBRAIC EQUATIONS: We find the solution to an equation with a single variable by rewriting the equation in the form f(x) = 0, and then using a root solver. The root-finding algorithm is known as the secant method. This method requires you to make an initial guess at the root. The algorithm then iterates from your guess until a true root is found. A warning is given if no root is found.

Let's try to find the solution to the equation

$$x^3 = e^x$$

First, we rewrite the equation in the form, f(x) = 0.

$$x^3 - e^x \qquad \text{or} \qquad f(x) = x^3 - e^x$$

Let's examine f(x) and determine the real (i.e., not complex) roots, which are the values of x that satisfy the equation

$$f(x) = 0 = x^3 - e^x$$

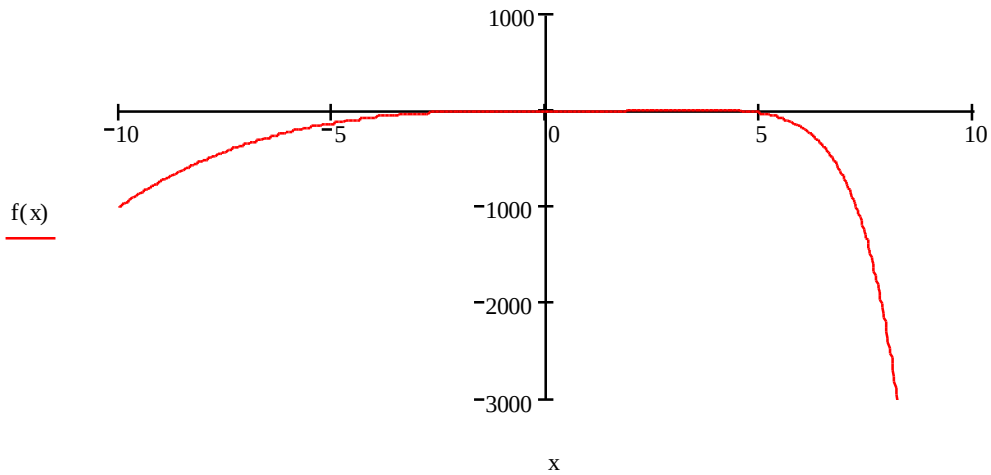
Because this is a cubic equation, we can anticipate that there might be multiple roots (i.e., more than one answer). However, it is not immediately clear how many real roots there are, since some of the roots may be imaginary. In any event, the root-finding algorithm will determine one root each time the algorithm is executed.

How can we ensure that distinctly different roots will be found each time? Generally, before beginning root finding we should get some idea of what the function "looks like". With MathCAD it is easy to do this by plotting the function. Looking at the plot, we can make reasonable guesses (estimates) for the value of the root we are trying to find.

OK. Now it's time for you to get typing! <u>Begin by defining the function $f(x) = x^3 - e^x$ and then plotting f(x) vs. x over the range -10<=x<=10.</u> Use the previous lessons as a guide for how to do this quickly and easily, without separately defining x.

For this exercise, you want a BIG graph. Are you surprised? You shouldn't be by now!

To observe where the function crosses the axes, f = 0, it is useful to select the "Crossed" option under "Axes Style".

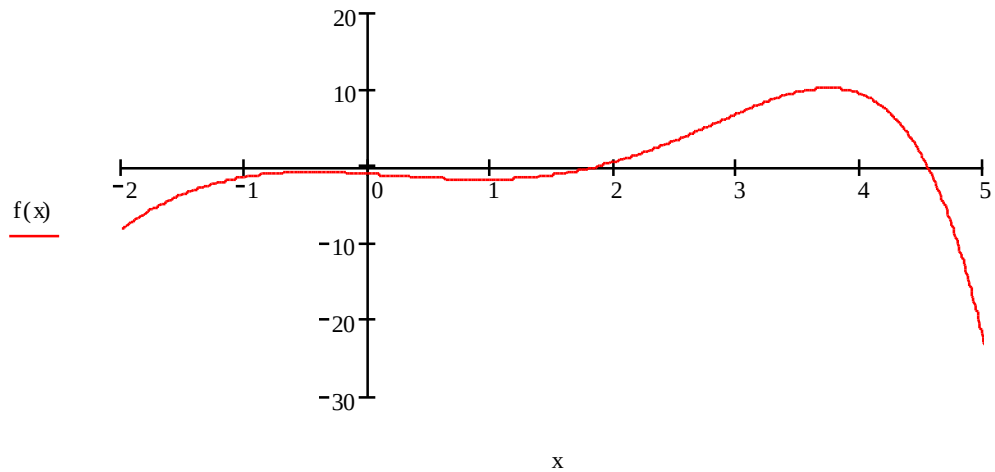


Looking at your graph, it seems that the function crosses from below the f = 0 axis into the positive region, and subsequently crosses back again into negative f region. This suggests the function has at least 2 real roots. You can find them using the 'root' function as follows. First, define a guess value of a root (say x = -5), and then use the root function.

x:-5
a:root(x^3 - e^x , x) or alternatively **a:root(f(x), x)**
a=

The result should be a = 1.857. Try different initial guesses. You should be able to find the second root at x = 4.536.

Notice that it is difficult to see where the curve crosses the axis. Recall from a previous lesson that you can quickly change the axis scaling. Change the x-axis scaling so that the graph plots from $-2 \le x \le +5$. This technique is often useful in looking for candidate guesses for roots.



More advanced use of *'root'*. If you check MathCAD's 'Help' facility, you will see that the *'root'* function can be used without having to make an initial guess. At least, the root solver still needs an initial guess, but you include it when you type in the command, rather than separately defining the guess. The difference is that when you separately define a guess, the root solver iterates toward a solution, and you cannot easily control the algorithm if there are several 'close' roots. When you use the extended form of the root solver, you force the algorithm to find a root between two prescribed values (if it exists). Try typing the following:

root(f(x),x,1,3)=
root(f(x),x,3,5)=

The first command returns 1.857, and the second line returns 4.536 (same answers as before). Try setting the root-solver limits to **root(f(x), x, 1, 5)** and see what happens. Look at your graph to interpret the error message.

Save your work to file **Mcad_Eq_01**

A SECOND EXAMPLE – ROOTS OF A POLYNOMIAL: Let's use the above procedure to find the roots to the polynomial

$$g(z) = z^3 - 10z + 2$$

Define the function g(z). Plot the graph with appropriate scales. Use the *'root'* function to show you can find the three real roots: -3.258, 0.201, and 3.057. If you think carefully *before* you start, you should be able to edit your work from the previous f(x) example to solve this example very quickly.

POLYROOTS: Approaching the problem as described above you should find three real roots (-3.258, 0.201, and 3.057). Another function exists for root-finding when the function is a polynomial such as this one. Define a column vector whose coefficients are the coefficients of the polynomial, beginning with the coefficient of the zero-order term in the first element and the coefficient of the highest order term in the last element. For this polynomial, the elements are (in order, lowest to highest) 2, -10, 0, 1. Get the following onto your worksheet:

$coef_0 := 2$
$coef_1 := -10$
$coef_2 := 0$
$coef_3 := 1$

Then use the MATHCAD function **polyroots(coef)** where coef is the vector of the coefficients.

polyroots(coef)=

Try this for the function g(z) described above and obtain the same roots noted above.

Also, try solving other polynomials. Plot them first, and verify you can use *'polyroots'* to determine the roots.

Save your work to file **Mcad_Eq_02** Open a blank worksheet in preparation for the next section

SOLVING LINEAR SIMULTANEOUS EQUATIONS: There are several ways to solve linear simultaneous equations. One method is to use a given/find solve block. This method is described later, since it can also be used to solve nonlinear equations.

The most common way of solving LINEAR simultaneous equations is to form the equations into matrices, and use matrix algebra. Consider the following four simultaneous equations:

$$10a + 3b - 4d = 12$$
$$a + 3b + 2c - d = -4$$
$$2b - 4c + 3d = 6$$
$$3a - 2b - c = 8$$

We want to write them in matrix form. We will use matrix ***B*** to hold the equation coefficients, and column matrices (vectors) ***X*** and ***Y*** to hold the variables and equation results respectively.

$$[B][X]=[Y]$$

where

$$[B]=\begin{bmatrix}10 & 3 & 0 & -4\\ 1 & 3 & 2 & -1\\ 0 & 2 & -4 & 3\\ 3 & -2 & -1 & 0\end{bmatrix};\qquad [X]=\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix};\qquad [Y]=\begin{bmatrix}12\\ -4\\ 6\\ 8\end{bmatrix}$$

We find the solution for ***X*** by multiplying the equation by the inverse of the ***B*** coefficient matrix:

$$X = B^{-1} * Y$$

Enter the four equations into MathCAD. Solve them using the previous method. Check your solution in 2 ways:

a) Use your calculator and manually enter the results into the equations. Check the equations are satisfied.

b) Perform (in MathCAD) the matrix multiplication B*X and check you get the correct results.

<u>SOLVING NON-LINEAR SIMULTANEOUS EQUATIONS (Given-Find):</u> The 'root' function works well for solving a single variable equation, f(x). Matrix algebra is very effective for solving several linear simultaneous equations. The following Given-Find method can solve a single-variable equation; several linear simultaneous equations; and several nonlinear simultaneous equations. What a powerful method!

Let's consider solving the following two simultaneous nonlinear equations

$$y = 6 - 2e^{-0.5x}$$
$$y = 4.5 + 0.2x^{1.2}$$

<u>The first step</u> is to define guess values for x and y. Recall from earlier that the best first thing to do is to plot the functions. This time we will plot the two functions on the same graph. The solutions are found where the graphs intersect.

Define the following two functions:

$$y1(x) = 6 - 2e^{-0.5x}$$
$$y2(x) = 4.5 + 0.2x^{1.2}$$

Plot the two functions on the same graph, with the x-axis defined in the range $0 \le x \le 10$.

Look at the graph, and notice that the first intersection is at about x = 1, and y = 4 (sometimes very approximate initial guesses are OK). Define your initial guess values for x and y as x = 1 and y = 4.

We now need to open what is called a ***Solve Block.*** To do this, type

Given<Enter>

where **<Enter>** means press the enter key. This opens the Solve-block. Unfortunately MathCAD does not highlight or otherwise mark the solve-block. At the moment the Solve-Block is just a white, blank area of the worksheet that extends below the ***Given*** as far down the worksheet as you want.

Below the Given (i.e., within the Solve-Block) enter the equations you want to solve. When you enter them, don't just hit the ***equals*** key. Instead, use **<CTRL>=**. The result is a dark **=** sign.

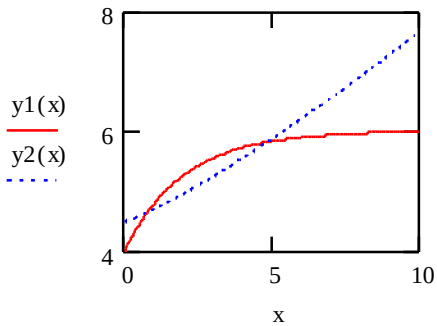
Finally, close the Solve-Block with a ***Find*** command, and look at the answers:

Find(x,y)=

Your worksheet should now look something like this (except your graph should be a LOT BIGGER!):

$y1(x) := 6 - 2 \cdot e^{-0.5 \cdot x}$ Define the equations and plot them

$y2(x) := 4.5 + 0.2 \cdot x^{1.2}$



$x := 1$ $y := 4$ Initial guesses for the first solution

Given Open Solve-Block

$y = 6 - 2 \cdot e^{-0.5 \cdot x}$

$y = 4.5 + 0.2 \cdot x^{1.2}$

$\text{Find}(x, y) = \begin{pmatrix} 0.786 \\ 4.65 \end{pmatrix}$

Change your guess values and confirm you can find the other result.

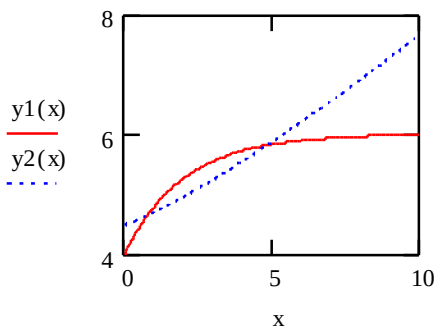
Answers: (0.786, 4.65) and (4.82, 5.82)

Did you notice that you had to type in each equation twice (once for graphing, and once to solve it)? Not only does that waste time, it also opens up the chance for more errors. Since you had already defined y1(x) and y2(x), you could have used those definitions in your Solve-Block. How?

Like this:

$y1(x) := 6 - 2 \cdot e^{-0.5 \cdot x}$ Define the equations and plot them

$y2(x) := 4.5 + 0.2 \cdot x^{1.2}$



$x := 1$ $y := 4$ Initial guesses for the first solution

Given Open Solve-Block

$y = y1(x)$ Use previous definitions of the equations

$y = y2(x)$

$\text{Find}(x, y) = \begin{pmatrix} 0.786 \\ 4.65 \end{pmatrix}$

Clean up your work. Put the matrix solution method on one page, and the Given-Find solution on a separate page.

Save your work to file **Mcad_Eq_03**

SUBMIT to your instructor:
Print the worksheet in file **Mcad_Eq_03** and submit the two printed pages.

Close MathCAD.
This introduction to Equation Solving is finished!