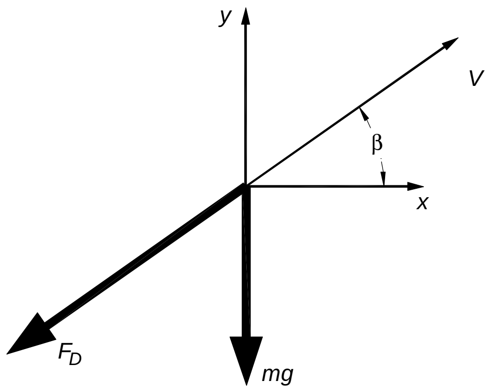
EM375 Project Handout
SIMULATION OF PROJECTILE MOTION

THEORY: For a projectile with air resistance, there are two forces of importance: the projectile’s weight, mg, and the drag due to air resistance, F_D. When the instantaneous angle of flight is β (the angle is measured as positive when it is above the horizontal, and negative when it is below), the force diagram is:



There are two equations of motion governing the flight of the projectile - one for the horizontal motion and one for the vertical motion:

$$m\frac{d^2x}{dt^2} = -F_D\cos(b) \tag{1}$$

and

$$m\frac{d^2y}{dt^2} = -mg - F_D\sin(b) \tag{2}$$

Now, the drag force depends on a number of parameters, including the speed of the projectile. The magnitude of the drag force is given by:

$$F_D = C_D A_f\, r_{AIR}\frac{V^2}{2} \tag{3}$$

where A_f is the frontal (silhouette) area of the projectile and C_D is a drag coefficient. For both the full-scale and model launchers a drag coefficient of $C_D = 0.5$ is appropriate.

Note: If we include a headwind (negative x-direction) of V_W, the drag force is increased and the x-component of the projectile’s speed has to be *decreased* by V_W in the following equations. If the wind is blowing in the positive-x direction (tailwind) the drag force is reduced.

The absolute speed, V, of the projectile is related to the components of its velocity by:

$$V = \sqrt{V_X^2 + V_Y^2} \tag{4}$$

We can relate the instantaneous angle, β, with the horizontal and vertical components of velocity as:

$$\sin(b) = \frac{V_Y}{V}$$
$$\cos(b) = \frac{V_X}{V} \qquad (5)$$

At this point in the handout, the governing equations are defined, and the drag force components are defined. What remains is to establish the coordinate system and initial conditions. The origin of the coordinate system will be taken at the *unstretched* point of the "sling". Using this point, the initial conditions become:

$$x = 0; \quad y = 0; \quad t = 0; \quad \frac{dx}{dt} = V_i \cos(b_i); \quad \frac{dy}{dt} = V_i \sin(b_i)$$

Now these equations can be rearranged and solved numerically using Mathcad.

PROCEDURE: Use the MathCAD Runge-Kutta ***"rkfixed"*** function to solve the two non-linear 2^{nd} order coupled differential equations established in the theory section. Run this model for both the full-scale and model slingshot launchers. Generate results for various launch angles and launch speeds.

You should work out how to obtain impact distance and time of flight for various different launch speeds and launch angles. For each launch angle you select, your aim is to determine several launch speed/impact distance pairs of values so that you can plot them on a separate graph.

TEST DATA: As with all good engineering simulations, you should check the output from your worksheet against known test data. For this exercise, we will consider baseball. What, approximately, is the speed of a ball when it leaves a hitter's bat? What is the weight of a baseball? What angle does it go up at as it leaves the bat? How far is a home run? Does your worksheet give sensible answers? How far would the ball go if the drag coefficient was zero (i.e., no air resistance)? Why don't they play baseball on the moon?

DATA REDUCTION: For the model launcher you need to determine the initial speed that was required in order for the valve ball to reach the measured distance. Using this speed (and other data provided by the other part of your team) you will be able to calculate the launcher efficiency, η. Assume the full scale launcher has the same efficiency.

For the full-scale launcher, use your MathCAD worksheet to create the distance vs. launch velocity plots that form the second part of your firing solution. It is best to use your worksheet to create a list (table) of values of initial speed and launch angle, with the resulting impact distance and time of flight. Then use a new MathCAD worksheet just to plot the results in the table.